Principles of Communication Systems

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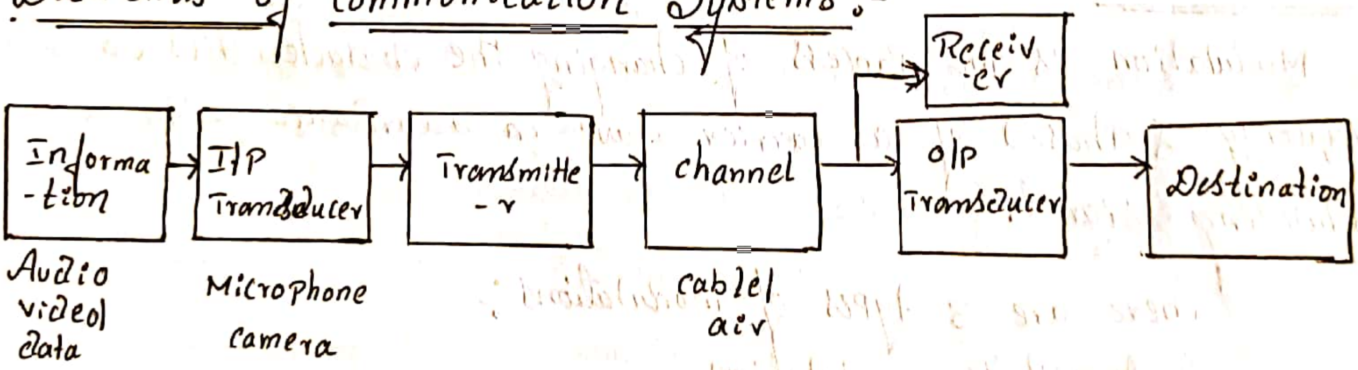
Module - 1 :-

Amplitude Modulation :-

Introduction :-

→ Communication is nothing but exchanging the information from one place to another place. There are two types of communication i) Local communication :- Local communication means face to face communication & ii) Remote communication :- Remote communication means transfering the information over a longer distance.
Ex :- Satellite & mobile communication.

Elements of communication Systems :-



→ The Purpose of the communication system is to transmit information signals through a communication channel.

→ first we need to give the information to the i/p Transducer, the infor-mation may be voice/ Speech, TV/ fascimile, Data from computer etc

→ Communication systems can be categorized by the types of information signals transmitted by the system. they are
1) Analog communication s/m's
2) Digital communication s/m's

→ Input Transducer :- It converts a physical signal from source to electrical, mechanical or electromagnetic signals suitable for communication.

→ Transmitter :- It has a function of processing the message into a form suitable for transmission over the channel.

→ Communication channel :- It provides a physical connection b/n the transmitter output & receiver input. The channel may be wired or wireless. Ex for wired channel is Telephone cable. Ex for wireless communication is Mobile & satellite communication.

→ Receiver :- It receives the transmitted signal from the channel.

→ Output transducer & destination :- At the output, message is recreated in it's original form, so that it is suitable for delivery to user destination.

Modulation :-

Modulation is the process of changing the characteristics (amplitude, frequency & phase) of a carrier wave in accordance with a modulating signal.

There are 3 types of modulations :-

i) Amplitude modulation

ii) Frequency modulation

iii) Phase modulation

Advantages or Need for Modulation :-

i) Reduces the height of Antenna :-

Height of antenna is a function of wavelength 'λ'. The minimum height of antenna is given by $\lambda/4$

i.e height of antenna $= \lambda/4 = \frac{c}{4f}$ $\quad \because \lambda = \frac{c}{f}$

Scanned with CamScanner

$C = 3\times10^{8}$ velocity of light

f = frequency

i) $f = 15\,KHz$

height of Antenna $= \lambda/4 = \frac{c}{4f} = \frac{3\times10^{8}}{4\times15\times10^{3}}$ = 5000 meters.

ii) $f = 1\,MHz$

height of Antenna $= \frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times10^{8}}{4\times1\times10^{6}}$ = 75 meters

from above two examples, it is clear that as the frequency increased, height of the Antenna is decreased.

ii) Avoids mixing of signals:-

All Audio signals ranges from 20Hz to 20KHz. The transmission of signals from various sources causes mixing of signals & it is difficult to seperate at the receiver end.

Thus modulating different signals sources by different carrier frequency avoids mixing of signals.

iii) Allows Multiplexing of signals:-

Multiplexing means Transmission of two or more signals simultaneously over the same channel.

Ex:- No: of TV channels operating simultaneously & No: of radio stations broadcasting the sig's.

iv) Increases range of communication:-

Modulation increases the frequency of signal to be radiated & thus increases the distance over which signal can be transmitted.

v) Allows Bandwidth adjustment:-

B.W of the modulated wave can be made smaller or larger than the original signal.

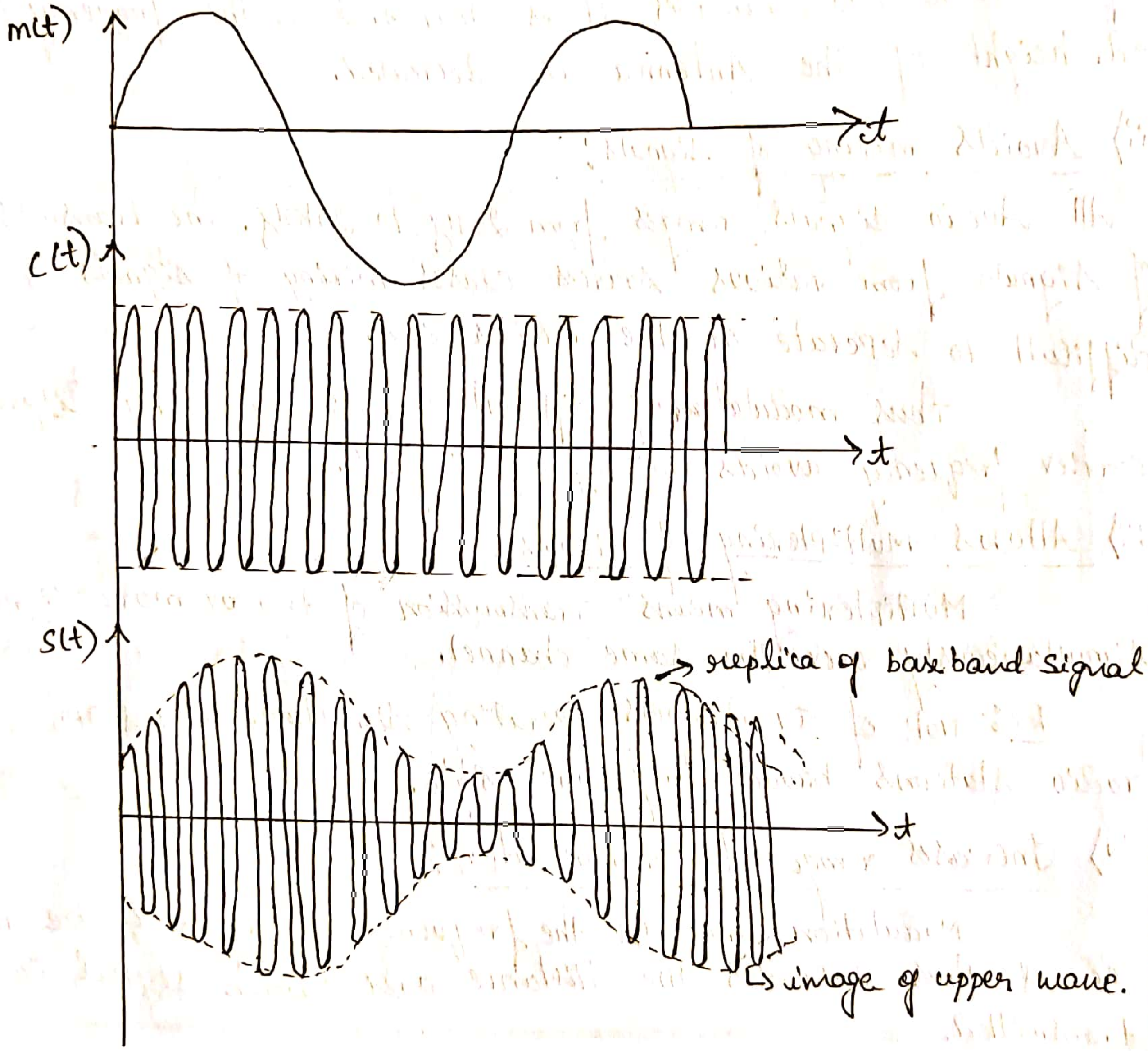
vi) Improves quality of reception:-

Modulation reduces the effect of noise, reduction of noise improves the quality of reception.

Amplitude modulation :-

The Process by which amplitude of a carrier is varied in accordance with the instantaneous modulating signal, keeping frequency & Phase constant.

Time domain & Frequency domain description of Am wave



Instantaneous value of modulating signal is given by.

$$m(t) = A_m \cos 2\pi f_m t \quad \text{—— (1)}$$

Carrier signal,

$$c(t) = A_c \cos 2\pi f_c t \quad \text{—— (2)}$$

where A_m & A_c are Amplitude of modulating & carrier slg
f_m & f_c are frequency of modulating & carrier slg

Eqn of Am is,

$$S(t) = A_{s(t)} \cos 2\pi f_c t$$

where $A_{s(t)} = A_c + m(t)$

$$\Rightarrow S(t) = [A_c + m(t)] \cos 2\pi f_c t$$

$$S(t) = [A_c + A_m \cos 2\pi f_m t] \cos 2\pi f_c t$$

$$= A_c\left[1 + \frac{A_m}{A_c} \cos 2\pi f_m t\right] \cos 2\pi f_c t$$

$$S(t) = A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t$$

where μ = modulation Index $= \frac{A_m}{A_c}$

$$S(t) = A_c\left[1 + \mu \frac{A_m}{A_m} \cos 2\pi f_m t\right] \cos 2\pi f_c t$$

$$S(t) = A_c\left[1 + \frac{\mu}{A_m} m(t)\right] \cos 2\pi f_c t$$

$$\boxed{S(t) = A_c[1 + K_a m(t)] \cos 2\pi f_c t}$$

K_a = amplitude sensitivity of the modulator

Frequency domain Description

The standard Am Expression is

$$S(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

$$S(t) = A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t$$

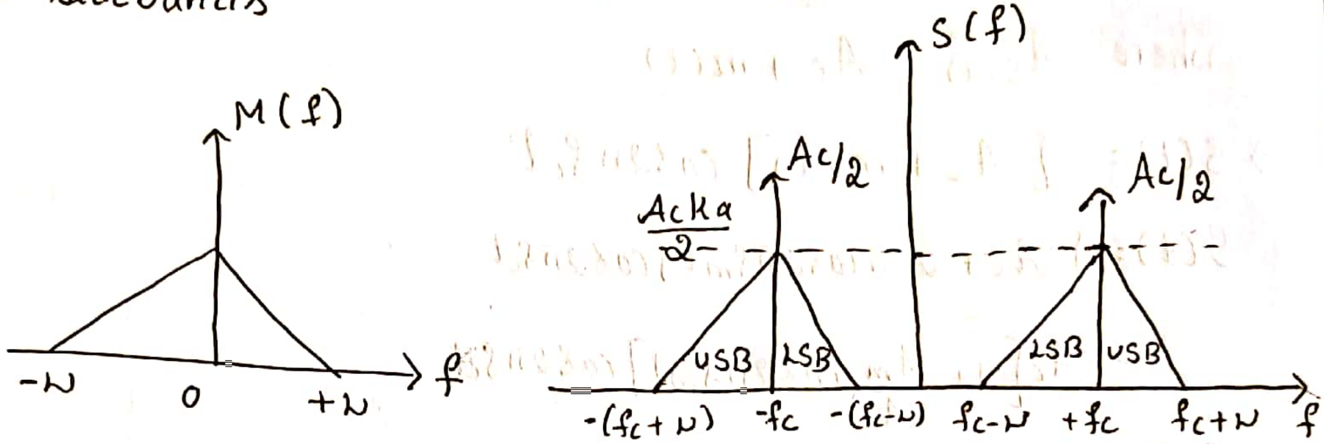
Taking Fourier transform on b.s, we get.

$$S(f) = A_c/2 [\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c K_a}{2}[M(f - f_c) + M(f + f_c)]$$

Note:- i) $\cos 2\pi f_c t \xleftrightarrow{F.T} \frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$

ii) $m(t)\cos 2\pi f_c t \xleftrightarrow{F.T} \frac{1}{2}[M(f-f_c)+M(f+f_c)]$

Thus, amplitude Spectrum of Am wave has carrier & two sidebands



Observation:-

i) Message Spectrum centered at $f=0$, Extending from $-W$ to W get translated to f_c.

ii) On Either side of $\pm f_c$, sidebands known as upper side band (USB) & Lower sideband (LSB) are present.

iii) Highest frequency component f_c+W is USB & Lowest frequency component f_c-W is LSB.

Transmission Bandwidth:-

The difference b/n USB & LSB frequencies is transmission Bandwidth

$$B.W = f_{USB} - f_{LSB}$$

$$B.W = (f_c+f_m)-(f_c-f_m)$$

$$B.W = \cancel{f_c}+f_m - \cancel{f_c}+f_m$$

$$\boxed{B.W = 2f_m}$$

$f_{USB} = f_c+W$ or f_c+f_m

$f_{LSB} = f_c-W$ or f_c-f_m

The Bandwidth required for transmission of Am wave is twice the frequency of modulating signal.

Single tone Modulation :-

In this type modulation, m(t) consists only single tone or frequency component.

$m(t) = A_m \cos 2\pi f_m t$, $\quad c(t) = A_c \cos 2\pi f_c t$

WKT, Am Expression is

$$S(t) = A_c[1+\mu \cos 2\pi f_m t] \cos 2\pi f_c t$$

$$S(t) = A_c \cos 2\pi f_c t + A_c \mu \cos 2\pi f_m t \cdot \cos 2\pi f_c t$$

$$S(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2}\left[\cos(2\pi f_c t + 2\pi f_m t) + \cos\{2\pi f_c t - 2\pi f_m t\}\right]$$

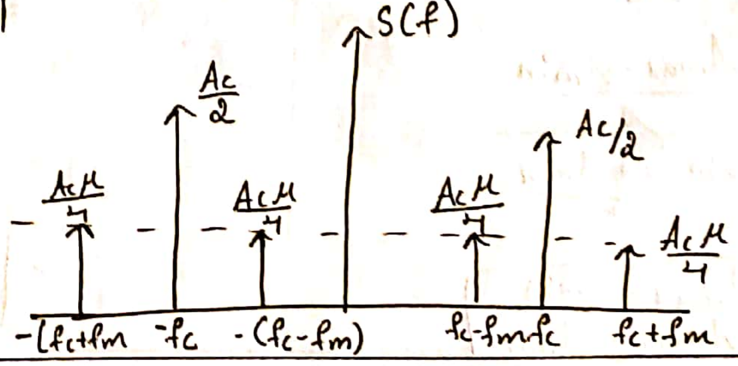
$$\left[\because \cos A \cdot \cos B = \tfrac{1}{2}[\cos(A+B) + \cos(A-B)]\right]$$

$$S(t) = A_c \cos 2\pi f_c t + \frac{A_c\mu}{2}\cos[2\pi(f_c+f_m)t] + \frac{A_c\mu}{2}\cos[2\pi(f_c-f_m)t]$$

Taking fourier Transform on B.S

$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{A_c\mu}{4}\left[\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m))\right] + \frac{A_c\mu}{4}\left[\delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m))\right]$$

The Spectrum of Am wave is



Modulation Index :- (μ)

It is the ratio of change in Amplitude of modulating signal to the amplitude of carrier wave. It is also known as modulation factor, modulation co-efficient or depth of modulation.

$$\boxed{\mu = \frac{A_m}{A_c}} \quad \text{or} \quad \boxed{\mu = k_a A_m}$$

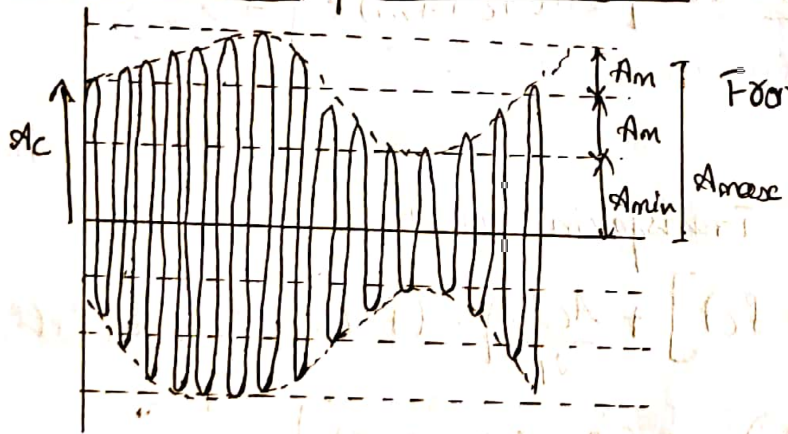
Percentage modulation Index is

$$\boxed{\% \mu = \frac{A_m}{A_c} \times 100}$$

Note:-

For Proper Amplitude modulation, A_m should be Always lesser than A_c i.e. $A_m < A_c$. If $A_m > A_c$, then distortion is introduced into the system.

μ in terms of A_{max} & A_{min} :-



From the fig, $A_{max} = A_m + A_m + A_{min}$

$A_{max} = A_{min} + 2A_m$

$2A_m = A_{max} - A_{min}$

$$\boxed{A_m = \frac{A_{max} - A_{min}}{2}}$$

WKT, $\mu = \frac{A_m}{A_c}$

$$\mu = \frac{\frac{A_{max} - A_{min}}{\cancel{2}}}{\frac{A_{max} + A_{min}}{\cancel{2}}}$$

$$\boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

$A_c = A_{max} - A_m$

$= A_{max} - \left[\frac{A_{max} - A_{min}}{2}\right]$

$= \frac{2A_{max} - A_{max} + A_{min}}{2}$

$$\boxed{A_c = \frac{A_{max} + A_{min}}{2}}$$

Expression for total Power in an Am wave:-

Am wave is given by,

$$S(t) = \underbrace{A_c \cos 2\pi f_c t}_{\text{Carrier}} + \underbrace{\frac{A_c \mu}{2} \cos[2\pi(f_c+f_m)t]}_{\text{USB}} + \underbrace{\frac{A_c \mu}{2} \cos[2\pi(f_c-f_m)t]}_{\text{LSB}}$$

It consists of 3 components: carrier, sidebands [USB & LSB]

thus, $P_T = P_C + P_{USB} + P_{LSB}$

$$P_T = \frac{A_c^2}{2R} + \frac{A_c^2\mu^2}{8R} + \frac{A_c^2\mu^2}{8R}$$

$$P_T = P_c + \frac{\cancel{2} A_c^2\mu^2}{\cancel{8}_4 R}$$

$$P_T = P_c + \frac{A_c^2\mu^2}{4R}$$

$$P_T = P_c + \frac{A_c^2}{2R}\left[\frac{\mu^2}{2}\right]$$

$$P_T = P_c + P_c\left[\frac{\mu^2}{2}\right]$$

$$\boxed{P_T = P_c\left[1+\frac{\mu^2}{2}\right]}$$

WKT, $P = VI$ (or)

$$P = \frac{V^2}{R}$$

$$P = \frac{V_{rms}^2}{R}$$

$$P = \frac{\left(\frac{V_m}{\sqrt{2}}\right)^2}{R}$$

$$P_C = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R}$$

$$\boxed{P_c = \frac{A_c^2}{2R}}$$

$$P_{USB} = P_{LSB} = \left(\frac{A_c\mu}{2\sqrt{2}}\right)^2 \Big/ R$$

$$\boxed{P_{USB} = P_{LSB} = \frac{A_c^2\mu^2}{8R}}$$

or

$$\boxed{P_{USB} = P_{LSB} = P_c\frac{\mu^2}{4}}$$

Note:- For perfect modulation, $\mu = 1$

Effective voltage & current for Am:-

voltage:-

$$P_T = P_c\left[1+\frac{\mu^2}{2}\right] \quad \Big|\ P = \frac{V^2}{R}$$

$$\frac{V_t^2}{\cancel{R}} = \frac{V_c^2}{\cancel{R}}\left[1+\frac{\mu^2}{2}\right]$$

$$V_t^2 = V_c^2\left[1+\frac{\mu^2}{2}\right]$$

Current:-

$$P_T = P_c\left[1+\frac{\mu^2}{2}\right] \quad \Big|\ P = I^2 \times R$$

$$I_t^2 \cancel{R} = I_c^2 \cancel{R}\left[1+\frac{\mu^2}{2}\right]$$

$$I_t^2 = I_c^2\left[1+\frac{\mu^2}{2}\right]$$

Taking square root on B.S

$$\boxed{V_t = V_c\sqrt{1+\mu^2/_2}}$$

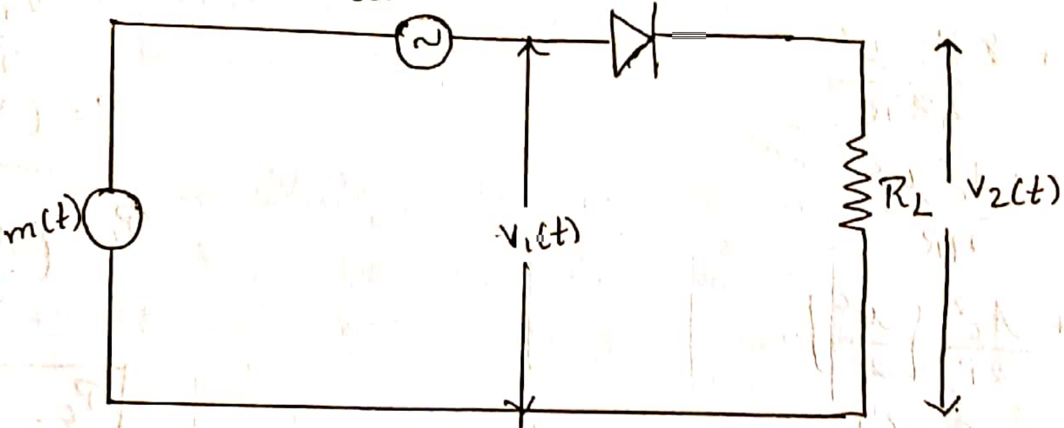
Taking square root on B.S

$$\boxed{I_t = I_c\sqrt{1+\frac{\mu^2}{2}}}$$

Generation of Am waves:-

Switching modulator is used for the generation of Am waves.

Switching modulator:-



circuit diagram of switching modulator is as shown here, diode act as ideal switch.

It is assumed that c(t) applied to the diode is large in amplitude i,e $A_c >> |m(t)|$

Total input to the diode is $v_1(t) = m(t) + c(t)$

$$\boxed{v_1(t) = m(t) + A_c\cos 2\pi f_c t} \qquad | \; c(t) = A_c\cos 2\pi f_c t \quad \text{—(1)}$$

output of diode $v_2(t)$ is

$$v_2(t) = \begin{cases} v_1(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases} \quad \text{—(2)}$$

$v_2(t)$ varies periodically b/n the values $v_1(t)$ & 0 at the rate equal to carrier frequency.

By assuming a modulating wave is weak compared to carrier wave, the non-linear behavior of the diode is replaced by an approximately equivalent linear-time varying operation; o/p of diode written as,

$$v_2(t) = v_1(t) \cdot g_p(t) \quad \text{—— (3)}$$

where $g_p(t)$ = Periodic rectangular train pulse $g_p(t)$ in Fourier Series form is

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c t (2n-1)\right]$$

i.e $g_p(t) = 1/2 + \underbrace{\frac{2}{\pi} \cos(2\pi f_c t)}_{n=1}$ + odd harmonic components —— (4)

Substituting Eq^n (4) in (3), we get

$$v_2(t) = \left[m(t) + A_c \cos 2\pi f_c t\right]\left[1/2 + \frac{2}{\pi} \cos(2\pi f_c t)\right]$$

$$v_2(t) = \frac{m(t)}{2} + \underbrace{\frac{2m(t)}{\pi} \cos 2\pi f_c t + \frac{A_c}{2} \cos(2\pi f_c t)}_{\text{wanted sig}} + \frac{2}{\pi} A_c \cos^2(2\pi f_c t)$$

The required AM wave centred at f_c is obtained by passing $v_2(t)$ through an ideal BPF having a center frequency 'f_c' & B.W = 2W or $2f_m$

output of BPF is $v_o(t)$

$$v_o(t) = \frac{2}{\pi} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t$$

$$= \frac{A_c}{2} \cos 2\pi f_c t \left[1 + \frac{4}{\pi A_c} m(t)\right]$$

$$v_o(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right] \cos 2\pi f_c t$$

—— (5)

Compare Eq^n (5) with the Standard Eq^n.

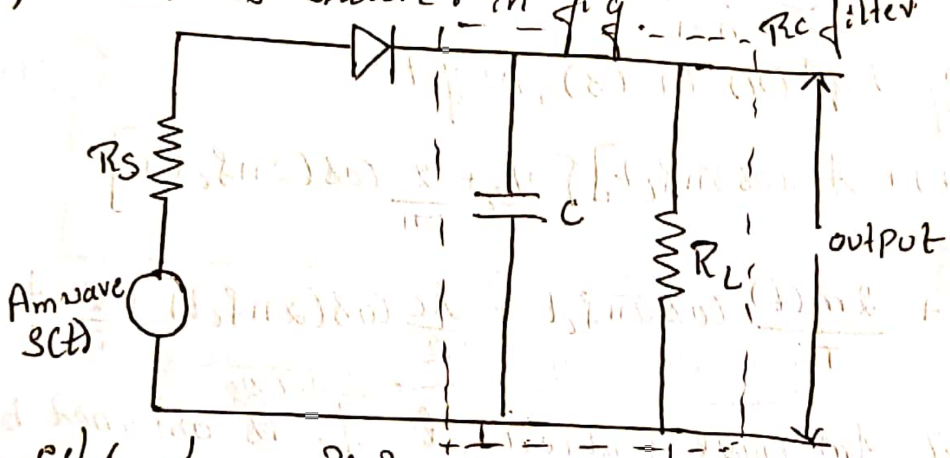
$$S(t) = A_c[1 + K_a m(t)] \cos 2\pi f_c t$$

$$K_a = \frac{4}{\pi A_c}$$

Envelope detector :-

→ Envelope detector is a simple & highly effective device used for demodulation of narrowband [$f_c >> f_m$]

→ Output of the envelope detector, follows the envelope of the input signal waveform exactly hence it is called Envelope detector, circuit is shown in fig.



→ It Consists of a diode & a resistor - capacitor [RC] filter. It is also known as "diode - detector circuit"

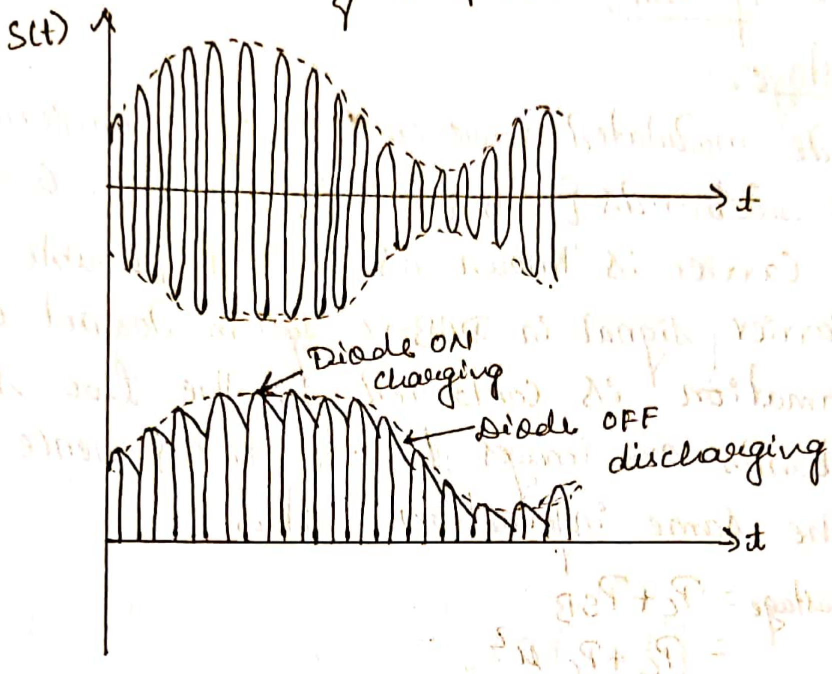
→ operation :-

i) on the positive half-cycle of the input signal S(t) i,e [0 to t1] diode is forward biased & capacitor C charges up rapidly to the peak value of the input signal.

ii) when input signal falls below this value, the diode becomes reverse-biased & capacitor C discharges slowly through the load resistance RL as shown in fig

→ The discharging process continuous untill the next positive half cycle.

→ When the input signal becomes greater than the peak value (A_c), the diode conduct again & the process is repealed.



Selection of RC time Constant

i) To rapidly charge the capacitor to the peak value of the input signal, the charging time constant $[CR_s]$ must be short compared with the carrier period. $1/f_c = T_c$

$$\text{i.e } \boxed{CR_s << 1/f_c}$$

ii) The discharging time constant CR_L should be large enough to ensure that the capacitor discharges slowly through the load resistance R_L

$$\text{i.e } \boxed{CR_L >> 1/f_c}$$

Advantages of Am waves:-

i) Am transmitter are less complex

ii) Am receivers are simple, detection is easy

iii) Am waves can travel a longer distance

Application of Am:-

i) Radio Broadcasting
ii) Picture transmission in a TV system.

Disadvantages of Am:-

i) Power wastage:-

Amplitude modulated wave consist of 3 components they are carrier & sidebands [USB & LSB]. Am wave consisting of 2 sidebands & carrier is known as DSB-FC [Double sideband-full carrier]. Carrier signal in DSB-FC system doesnot convey any information. Information is contained in the two sidebands only. But sidebands are images of each other & hence both of them contain the same information, thus

$$P_{wastage} = P_C + P_{SB}$$
$$= P_C + P_C \frac{\mu^2}{4}$$
$$= P_C \left[1 + \frac{\mu^2}{4}\right]$$

ii) Bandwidth inefficient system:-

B.W required for Am transmission is $2f_m$, this is due to the simultaneous transmission of both the sidebands. out of which only one sideband is sufficient to convey all the information. thus the B.W of DSB-FC is double than actually required.

iii) Effect of noise:-

when the Am travels from transmitter to receiver over a communication channel, noise gets added to it. The noise changes the amplitude of the envelope of AM ~~Leades~~ Leads to information contamination in Am. Hence, the Performance of Am is very Poor in Presence of Noise.

Transmission Efficiency of an AM wave:-

Transmission Efficiency is defined as the ratio of the Power carried by sidebands to the total transmitted Power is called transmission Efficiency 'η' & it given by

$$\eta = \frac{P_{USB} + P_{LSB}}{P_T}$$

WKT,

$$P_T = P_c\left[1 + \frac{\mu^2}{2}\right] \quad \& \quad P_{USB} = P_{LSB} = \frac{\mu^2 A_c^2}{8R}$$

$$\eta = \frac{\frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}}{P_c\left[1 + \frac{\mu^2}{2}\right]}$$

$$= \frac{\frac{\cancel{2}^{1}\mu^2 A_c^2}{\cancel{8}^{4} R}}{P_c\left[1 + \frac{\mu^2}{2}\right]} = \frac{\frac{\mu^2}{2}\left[\frac{A_c^2}{2R}\right]}{P_c\left[1 + \frac{\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2}{2} \times \cancel{P_c}}{\cancel{P_c}\left[1 + \frac{\mu^2}{2}\right]} = \frac{\mu^2/\cancel{2}}{\frac{2 + \mu}{\cancel{2}}}$$

$$\boxed{\eta = \frac{\mu^2}{\mu^2 + 2}}$$

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MODULE-1

Double Side Band-Suppressed carrier Modulation [DSB-SC]

To overcome the drawback of power wastage in an AM wave (DSB-FC) an DSB-SC method is used.

The conventional AM wave in which the carrier is suppressed is called DSB-SC modulation. Since carrier component doesnot contain any information, it is suppressed. Thus DSB-SC is a method of transmission only the two sidebands without the carrier.

In DSB-SC no power is waster on the carrier and the saved power can be put into the sidebands for stronger signals over longer distances.

Time and Frequency domain description

Time domain description

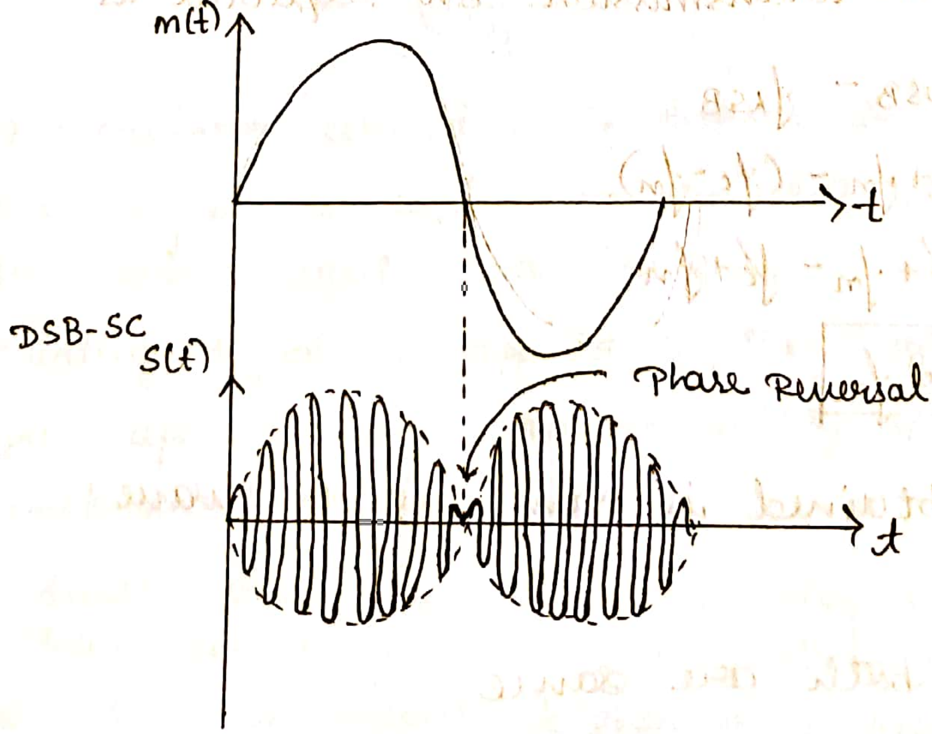
The standard DSB-SC wave is

$$\langle s(t) = c(t)\, m(t) \rangle$$

$$s(t) = m(t) A_c \cos 2\pi f_c t \qquad \text{where, } c(t) = A_c \cos 2\pi f_c t$$

where, $m(t)$ is modulating/message signal
$c(t)$ is carrier wave.

DSB-SC modulated wave undergo a phase reversal whenever the modulating signal $m(t)$ crosses a zero.



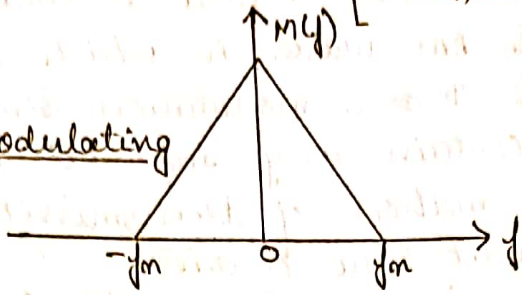
Frequency domain description:

w.k.t $s(t) = m(t) A_c \cos 2\pi f_c t$

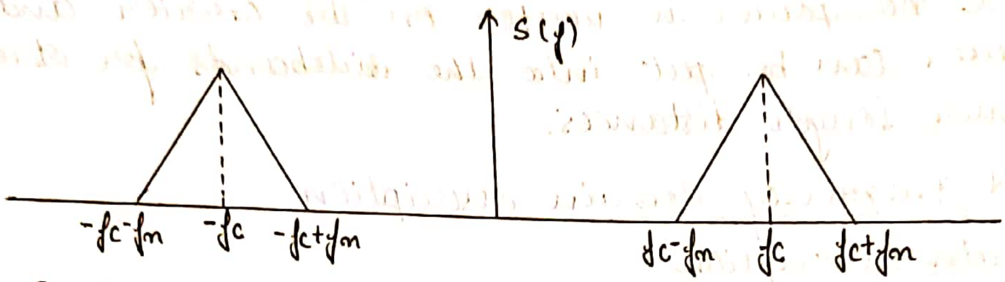
Taking fourier transform on both sides

$$S(f) = \frac{A_c}{2}\left[M(f+f_c) + M(f-f_c)\right]$$

$$\left[\because m(t)\cos 2\pi f_c t \xleftrightarrow{FT} \frac{1}{2}\left[M(f+f_c) + M(f-f_c)\right]\right]$$



Spectrum of Modulating Signal.



Spectrum of DSB-SC modulated wave.

- Dotted line at $\pm f_c$, indicates that carrier wave is suppressed.
- On either side of $\pm f_c$, two sidebands designated as upper (f_c+f_m) and lower (f_c-f_m) sidebands are present.
- The minimum transmission BW required is

$$BW = f_{USB} - f_{LSB}$$
$$= f_c + f_m - (f_c - f_m)$$
$$= \cancel{f_c} + f_m - \cancel{f_c} + f_m$$
$$\boxed{BW = 2f_m}$$

Bandwidth obtained is same as AM wave

Note:

W & f_m both are same.

Generation of DSB-SC waves:

DSB-SC modulated wave simply a product of message signal and the carrier signal. a device for achieving this is called a product modulator.

Two forms of product modulator used for DSB-SC generation are

1) Balanced Modulator
2) Ring Modulator

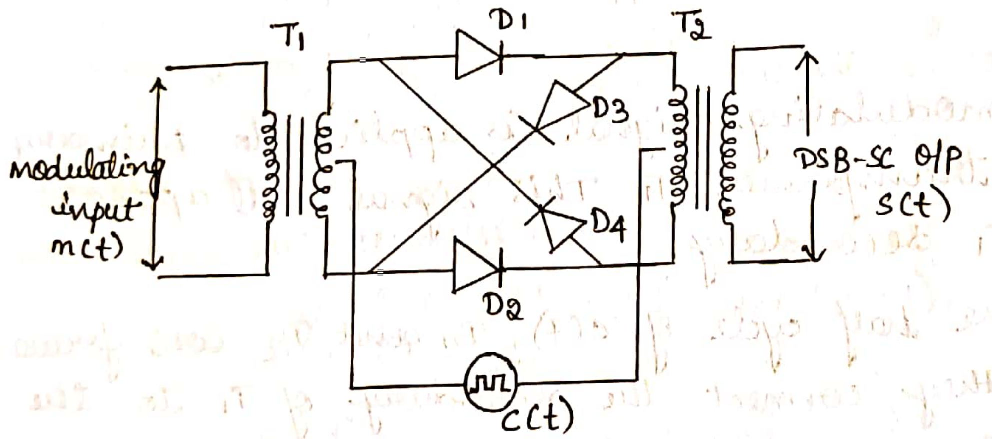
Ring Modulator:

Ring modulator is the most popular and widely used efficient modulator for generating DSB-SC.

It is also known as lattice or double balanced modulator.

It consists of

1) An input transformer, T_1
2) Four diodes connected in ring form
3) An output transformer, T_2.



Ring modulator circuit is as shown in above figure

* carrier signal is applied to the center taps of the input and output transformers.
* Modulating signal is applied to the input transformer T_1.
* Output appears across the secondary of the output transformer T_2.
* The diodes are connected in the ring act like switches and their switching action is controlled by the carrier signal as it is usually higher in frequency.

Operation:

Case 1) when $m(t)=0$

In the +ve half cycle of the carrier signal, diode D_1 and D_2 are forward biased & D_3 and D_4 are reverse biased. current divides equally in the upper and lower portions of the primary windings of T_2 but opposite to each other i.e they are equal in magnitude Hence they cancel each other. thus no output is produced at secondary winding of T_2, carrier is suppressed.

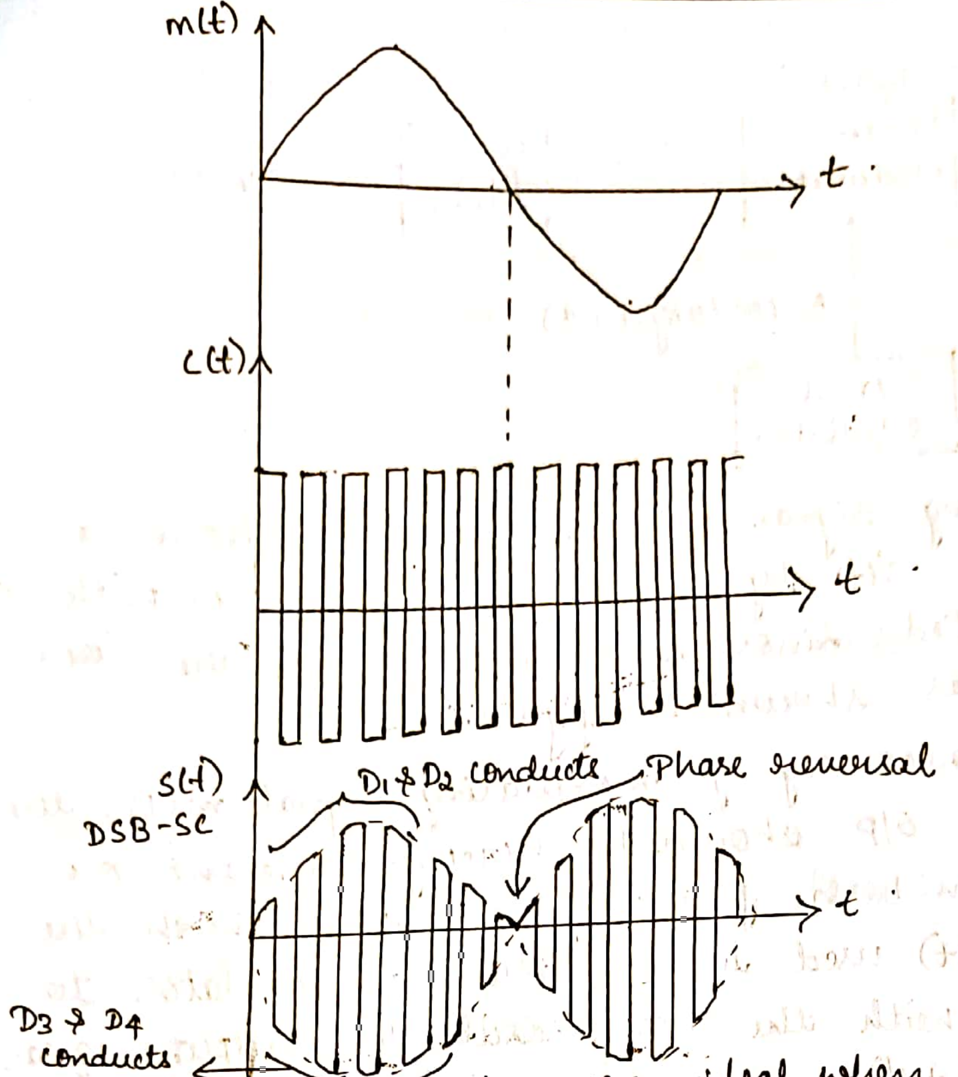
During -ve half cycle of $c(t)$, Diode D_1 and D_2 are reverse biased and D_3 and D_4 are forward biased. Here also current divides equally at T_2 but opposite to each other. Hence they cancel each other. As a result no output is produced at secondary winding of T_2, thus carrier is suppressed.

Case 2)

When the modulating signal is applied to primary winding of transformer T_1. This signal will appear across the T_1 secondary.

In the +ve half cycle of $c(t)$, D_1 and D_2 are forward biased and they connect the secondary of T_1 to the primary of T_2. As a result output at secondary of T_2 is modulating signal, i.e $(O/P = m(t))$.

In the -ve half cycle of $c(t)$, D_3 and D_4 are forward biased and they will connect the secondary of T_1 to the primary of T_2 with reverse connections. This results in 180° phase shifts in the modulating signal. i.e $(O/P = -m(t))$.



The ring modulator is more ideal when carrier is a square wave

Thus c(t) representation is

$$c(t) = 4/\pi \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c t\,(2n-1)\right]$$

$$c(t) = 4/\pi \left[\cos 2\pi f_c t - \underbrace{\frac{1}{3}\cos 2\pi f_c t\,(3) + \cdots\cdots}_{\text{Odd harmonics}}\right]$$

W.K.T for DSB-SC

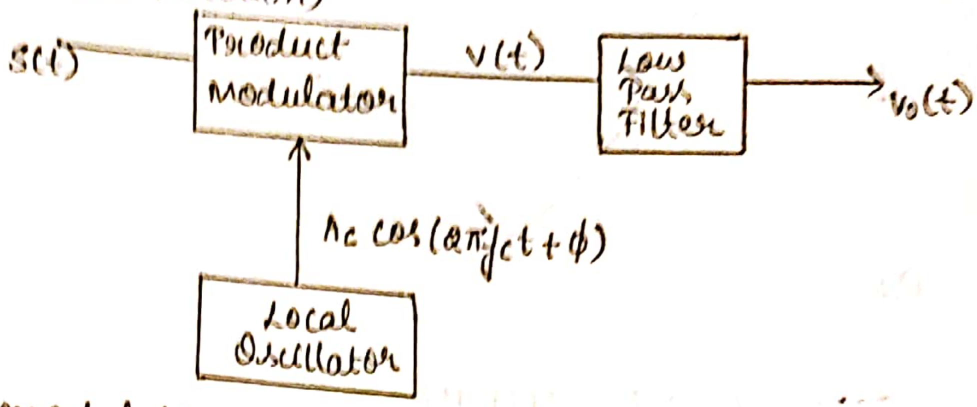
$$s(t) = m(t)\, c(t)$$

$$s(t) = m(t)\left[\frac{4}{\pi} \cos 2\pi f_c t\right]$$

Output of Band pass filter is given by,

$$\left\langle s'(t) = 4/\pi\, m(t) \cos 2\pi f_c t \right\rangle$$

Coherent Detection of DSB-SC wave:
(Synchronous Detection)



The modulating signal m(t) is recovered from a DSB-SC wave s(t) by first multiplying s(t) with a locally generated sinusoidal wave & then the low pass filter as shown in figure.

For faithful recovery of modulating signal m(t), the local oscillator O/P should be exactly coherent or synchronized in both frequency and phase with the carrier wave c(t) used in the product modulator to generate $V_o(t)$ with the local oscillator output equal to $\cos(2\pi f_c t + \phi)$.

From the above figure, output of product modulator is

$$v(t) = s(t) \cos(2\pi f_c t + \phi)$$

W.K.T

$$s(t)_{DSB\text{-}SC} = m(t)\, c(t)$$

$$= m(t)\, A_c \cos 2\pi f_c t$$

$$v(t) = m(t)\, A_c \cos 2\pi f_c t \, \cos(2\pi f_c t + \phi)$$

$$\cos A \cdot \cos B = \tfrac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$= \frac{A_c m(t)}{2}[\cos(4\pi f_c t + \phi) + \cos(-\phi)]$$

$$v(t) = \underbrace{\frac{A_c}{2} m(t) \cos(4\pi f_c t + \phi)}_{\text{unwanted signal}} + \underbrace{A_c/2 \; m(t) \cos\phi}_{\text{scaled version of message signal.}}$$

When V(t) passes through the LPF, it removes the unwanted ~~fs~~ term in the output of product modulator

i.e $V_o(t) = \frac{A_c}{2} m(t) \cos\phi$

Thus demodulated signal $V_o(t)$ is therefore proportional to m(t).

where ϕ is phase error, which is constant

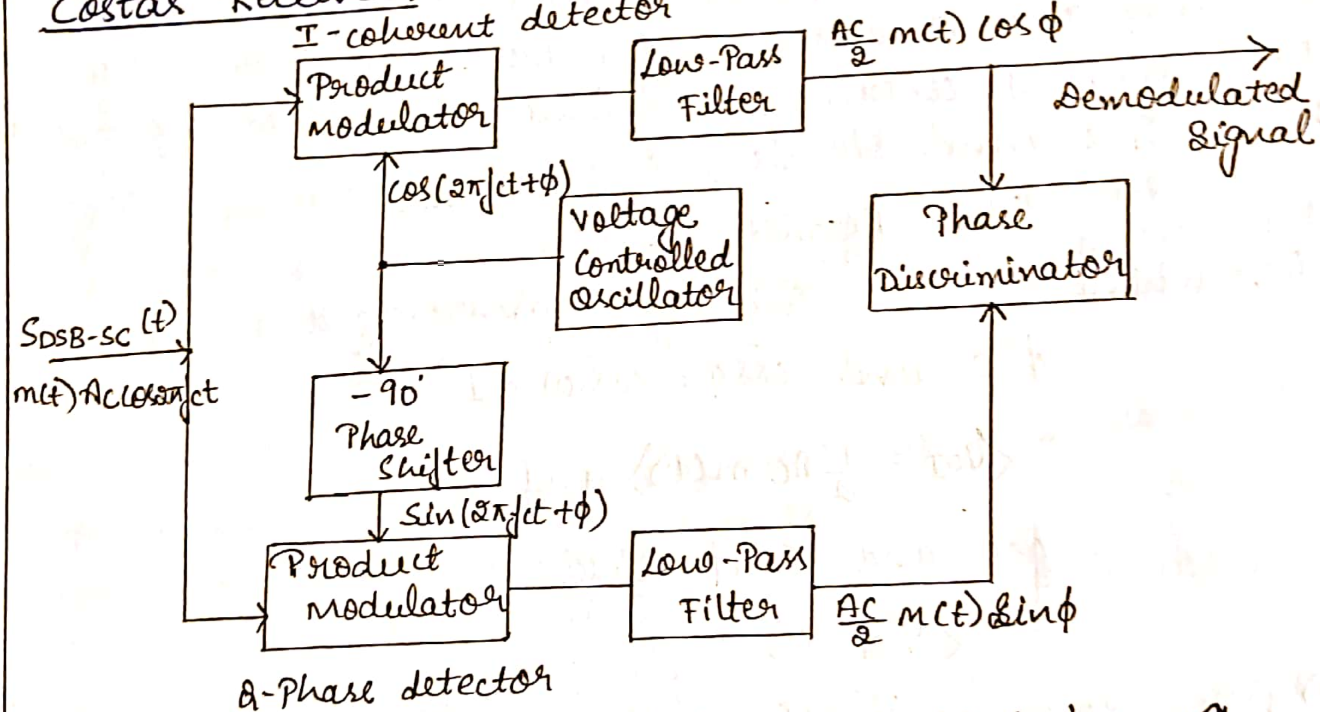
when $\phi = 0$, amplitude of $V_o(t)$ is maximum.

when $\phi = \pm\frac{\pi}{2}$, amplitude of $V_o(t)$ is minimum.

$\Downarrow$

It represents the "Quadrature Null Effect" of coherent detector.

Costas Receiver:



* The costas loop is a method of obtaining a practical synchronous receiver system, suitable for demodulating DSB-SC waves.

* The receiver consists of two coherent detectors supplied with the same input signal i.e DSB-SC waves $A_c m(t) \cos 2\pi f_c t$ but individual local oscillator signals that are in-phase quadrature with respect to each other. (i.e the local oscillator signal supplied to the product modulators are

90° out of phase)

* The frequency of the local oscillator is adjusted to be the same as the carrier frequency 'f_c'.

* The detector in the upper path is referred to as In-phase coherent detector or I-channel and that in the lower path is referred to as the Quadrature-phase coherent detector or Q-channel.

* These two detectors are coupled together to form a Negative feedback system designed in such a way as to maintain the local oscillator synchronous with the carrier wave.

Operation:

1) When local oscillator signal is of the same phase as the carrier wave $A_c \cos 2\pi f_c t$ used to generate the incoming DSB-SC wave. Under these conditions, the I-channel O/P contains the desired demodulated signal $m(t)$, where Q-channel O/P is zero.

$$V_{OI} = \frac{A_c}{2} m(t) \cos\phi$$

i.e whenever the carrier is synchronized

$$\phi = 0 \text{ and } \cos\phi = \cos(0) = 1$$

$$\langle V_{OI} = \frac{1}{2} A_c\, m(t) \rangle \text{ and}$$

$$\phi = 0 \text{ and } \sin\phi = \sin(0) = 0$$

$$\langle V_{OQ} = 0 \rangle$$

2) When local oscillator phase changes by a small angle 'ϕ' radians, the I-channel output will remain unchanged, but Q-channel produces some O/P which is proportion to $\sin\phi$.

* The O/P of I and Q-channels are combined in phase-discriminator (which consists of a multiplier followed by a LPF), a dc control signal is obtained, that automatically corrects for local phase errors in the voltage controlled-Oscillator (VCO).

Vestigial Sideband Modulation [VSB]:-

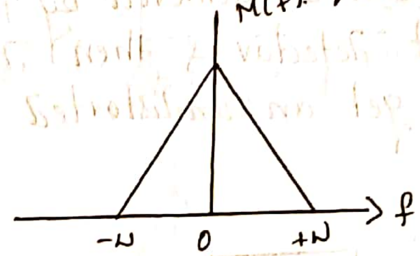
→ The SSB modulation is not appropriate way of modulation when the message signal contains significant components at extremely low frequencies.

→ In such cases the upper & lower sidebands meet at the carrier frequency & it is difficult to isolate one sideband. To overcome this difficulty the modulation technique known as vestigial sideband modulation [VSB] is used.

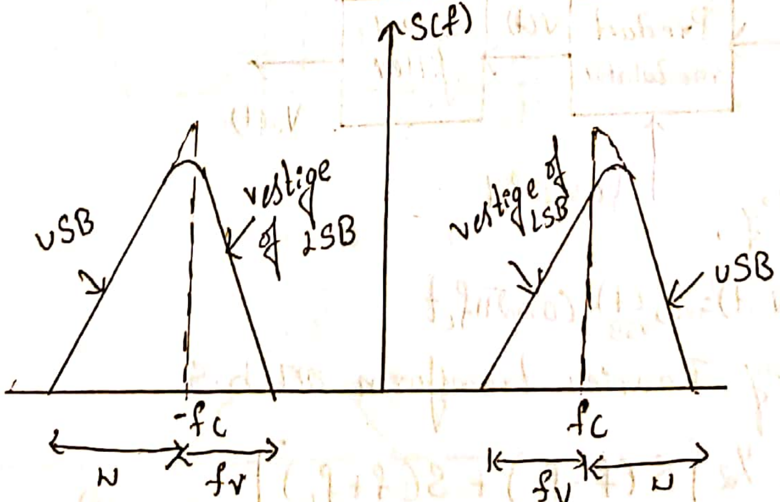
→ In VSB, one sideband is passed almost completely whereas just a trace / vestige of the other sideband is retained. VSB is the compromise between SSB & DSB-SC modulation. It is widely used in television transmission.

Frequency domain description:-

Spectrum of VSB & Modulating signal m(t) is shown in fig.



Spectrum of msg sig

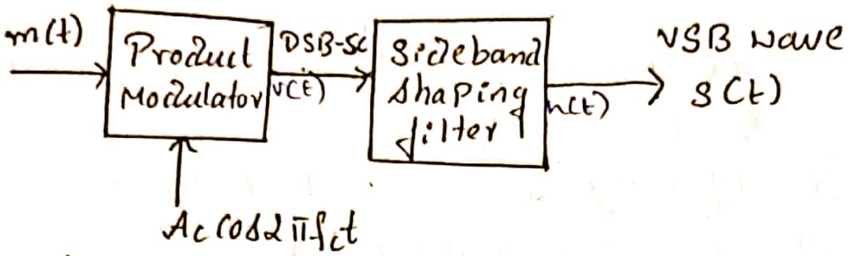


Note:-

$BW = W + f_v$

Generation of VSB modulated wave:-

VSB modulated wave is generated by passing DSB-SC modulated wave through a sideband shaping filter, as shown in fig.



From the fig,

$$s(t) = v(t) * h(t)$$

Taking Fourier transform on B.S

$$S(f) = V(f)\,H(f)$$

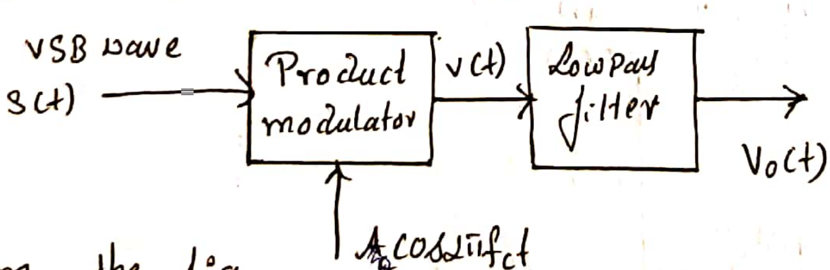
$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] H(f)$$

$$v(t) = S_{DSB\text{-}SC}(t) = A_c m(t) \cos 2\pi f_c t$$

$$V(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

Demodulation of VSB Modulated wave:-

Demodulation of VSB can be achieved by passing VSB wave s(t) through a coherent detector & then detector output passes through LPF to get an undistorted version of the original signal m(t).



From the fig,

$$v(t) = S_{VSB}(t) \cos 2\pi f_c t$$

Taking Fourier transform on b.s

$$V(f) = \frac{1}{2}\left[S(f-f_c) + S(f+f_c)\right] \quad \text{———} \; (*)$$

WKT, $S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] H(f)$

$$S(f-f_c) = \frac{A_c}{2}\left[M(f-2f_c) + M(f)\right] H(f-f_c) \quad \text{— (1)}$$

$$S(f+f_c) = \frac{A_c}{2}\left[M(f) + M(f+2f_c)\right] H(f+f_c) \quad \text{— (2)}$$

Substituting Eq^n (1) & (2) in (*)

$$V(f) = \tfrac{1}{2}\left[S(f-f_c) + S(f+f_c)\right]$$

$$= \frac{1}{2}\left[\left[\frac{A_c}{2} M(f-2f_c) + M(f)\right] H(f-f_c) + \left[\frac{A_c}{2} M(f) + M(f+2f_c)\right] H(f+f_c)\right]$$

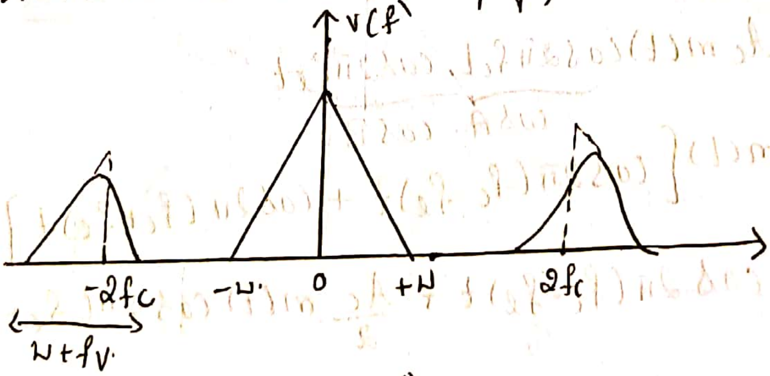
$$V(f) = \frac{A_c}{4} M(f-2f_c) H(f-f_c) + \frac{A_c}{4} M(f) H(f-f_c) + \frac{A_c}{4} M(f) H(f+f_c) + \frac{A_c}{4} M(f+2f_c) H(f+f_c)$$

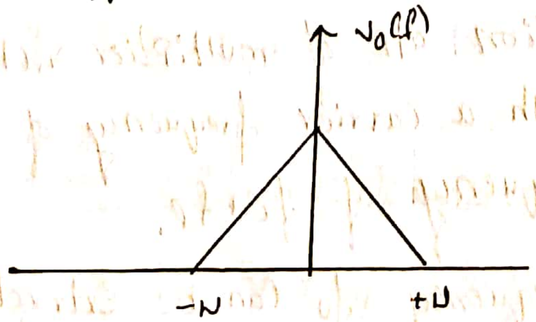
$$V(f) = \frac{A_c}{4}\left[H(f-f_c) + H(f+f_c)\right] M(f) + \frac{A_c}{4}\left[M(f-2f_c) H(f-f_c) + M(f+2f_c) H(f+f_c)\right]$$

Then Spectrum Passing through LPF,

$$V_o(f) = \frac{A_c}{4}\left[H(f-f_c) + H(f+f_c)\right] M(f)$$

The Spectrum is as shown in fig,





W & for both are same.

VSB – Transmission of Analog & Digital Television

→ Single Sideband transmission is not possible since one Sideband cannot be suppressed fully by physically realizable filters. Therefore some part of lower sideband close to picture carrier is also transmitted.

→ This part of LSB is mainly used to accommodate roll-off characteristics of filters. This avoids the attenuation of low frequencies near picture carrier. This type of transmission in which complete upperside & small part of LSB is transmitted is called VSB.

→ The channel Bandwidth used is 6MHz, which consists of VSB modulated video signal & sound signal as shown in fig, the picture carrier is at 55.25MHz & sound carrier is at 59.75MHz

→ Based on following 2 factors VSB modulation is used.

i) The video signals has large bandwidth & significant low frequency content.

ii) The demodulation circuit must be simple & cheap. So we have to use Envelope detector, which requires addition of carrier in VSB modulated slg.

→ To fulfil above factors, the transmitted wave must have high power level, so it's not a VSB modulation, instead VSB filter is inserted in each receiver, where power levels are low. The overall performance is the same as conventional VSB modulation except for some wasted power & B.W

→ VSB Technique can also be applied to digital signals for high definition [HD] tv signals with following factors.

i) The transmitted signal must be compatible in terms of B.W

Principles of Communication Systems
Module - 2
Angle Modulation

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Angle Modulation:

It is a process in which either frequency or phase [i.e angle] of the carrier is varied in accordance with the instantaneous amplitude of the message signal, keeping amplitude of the carrier wave constant.

Angle modulated wave can be expressed as,

$$s(t) = A_c \cos[\theta(t)]$$

where,

A_c = carrier amplitude, maintained constant

$\theta(t)$ = Angular argument which is varied in proportion with the message signal $m(t)$.

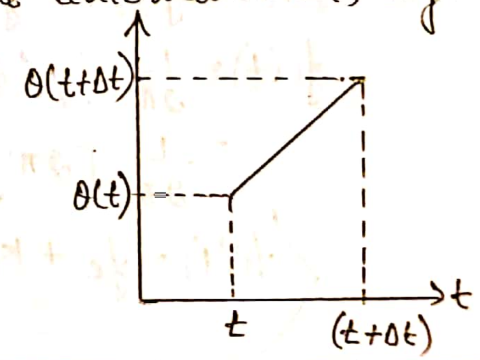
Depending ~~on~~ the $\theta(t)$ changes, there are two types of angle modulation.

1) Frequency modulation
2) Phase Modulation.

Instantaneous frequency of Angle modulated wave $f_i(t)$:

* The variation of $\theta(t)$ due to $m(t)$ can be expressed mathematically based on the type of angle modulation.
* $\theta(t)$ changes by 2π radians to complete one oscillation, if $\theta(t)$ is increased monotonically with time as shown in figure:
* then the average frequency over the interval $(t+\Delta t)$ is given by

$$f_{avg} = f_{\Delta t}(t) = \frac{\theta(t+\Delta t) - \theta(t)}{\Delta t}$$



Then the instantaneous frequency of the angle modulated wave $s(t)$ is given by.

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$= \lim_{\Delta t \to 0} \left[\frac{\theta(t+\Delta t) - \theta(t)}{2\pi \Delta t} \right]$$

$$\boxed{f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}}$$

<u>Note</u>: For an unmodulated carrier, the angle $\theta(t)$ is

$$\langle \theta(t) = 2\pi f_c t + \phi \rangle$$

<u>Phase modulation:</u>

PM is defined as the form of angle-modulation in which the angular argument '$\theta(t)$' is varied linearly with the message signal '$m(t)$' as given below:

i.e $\quad \theta(t) = 2\pi f_c t + K_p m(t)$

where, $K_p \rightarrow$ phase sensitivity of the modulator, which is constant.

W.K.T $\quad S_{\text{angle modulated}}(t) = A_c \cos[\theta(t)]$

Then, the phase modulated wave $S_{PM}(t)$ is given by.

$$S_{PM}(t) = A_c \cos[2\pi f_c t + K_p m(t)]$$

<u>Instantaneous frequency $f_i(t)$ in PM wave:</u>

W.K.T $\quad f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$

In PM, $\theta(t) = 2\pi f_c t + K_p m(t)$

Substituting in $f_i(t)$, we get

$$f_i(t) = \frac{1}{2\pi} \frac{d}{dt} [2\pi f_c t + K_p m(t)]$$

$$= \frac{1}{2\pi} \left[2\pi f_c + K_p \frac{d}{dt} m(t)\right]$$

$$\left\langle f_i(t) = f_c + \frac{K_p}{2\pi} \frac{d\,m(t)}{dt} \right\rangle$$

Thus in phase modulation, instantaneous frequency $f_i(t)$ varies linearly with the derivative of $m(t)$.

Frequency Modulation:

It is the form of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the message signal $m(t)$

i.e $\langle f_i(t) = f_c + k_f\, m(t) \rangle$

where, k_f = Frequency sensitivity, which is constant.

Angular argument in FM [$\theta(t)$]:

W.K.T $f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$

$$d\theta(t) = 2\pi f_i(t)\, dt$$

Integrate on B.S w.r.t t

$$\int_0^t d\theta(t) = \int_0^t 2\pi f_i(t)\, dt$$

$$\theta(t) = 2\pi \int_0^t f_i(t)\, dt$$

substituting $f_i(t) = f_c + k_f m(t)$, we get

$$\theta(t) = 2\pi \int_0^t [f_c + k_f m(t)]\, dt$$

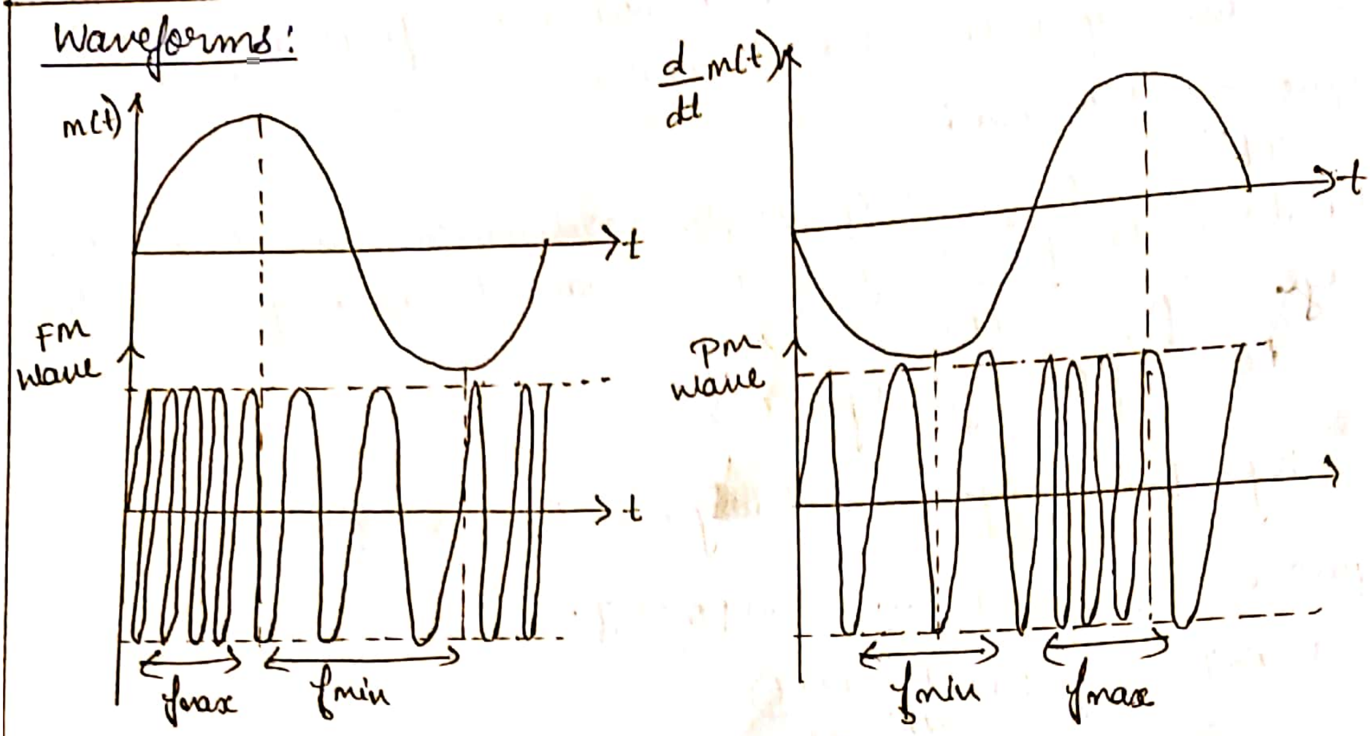
$$\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt$$

Thus, frequency modulated wave $S_{FM}(t)$ is given by,

$$\left\langle S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right] \right\rangle$$

Thus in frequency modulation, angular argument is varied linearly with the integral of the $m(t)$.

Waveforms:



Relationship between FM and PM : OR

Generation of i) FM using PM
ii) PM using FM.

The time domain expression for FM and PM wave are

FM wave : $S_{FM}(t) = A_c \cos[2\pi f_c t + 2\pi K_f \int_0^t m(t)dt]$

Pm wave : $S_{PM}(t) = A_c \cos[2\pi f_c t + K_p m(t)]$

comparing above two expressions, we can conclude that FM wave is same as PM wave, if m(t) in PM wave is replaced by $\int_0^t m(t)\,dt$.

i) Generation of FM wave using PM [Phase modulator]

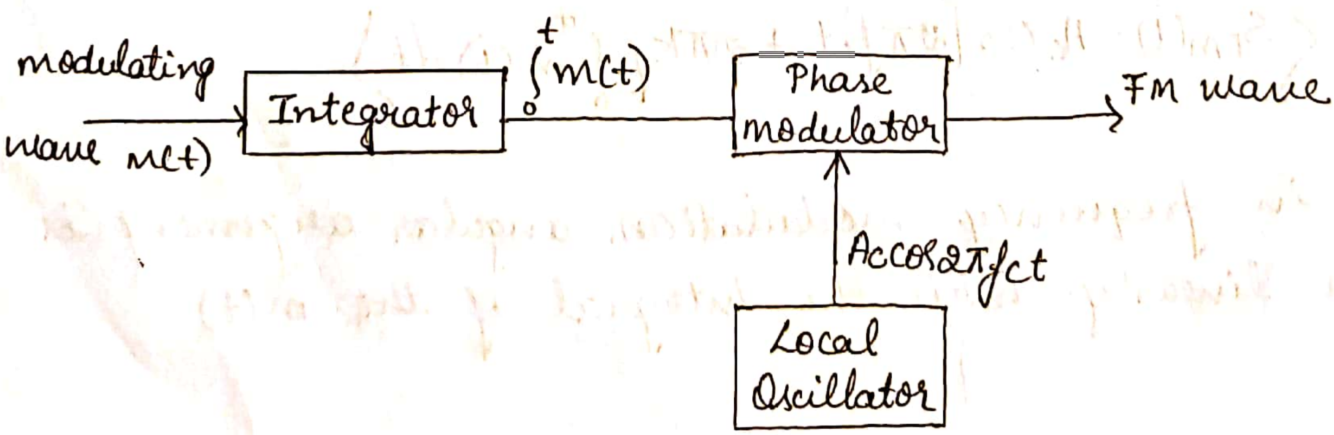


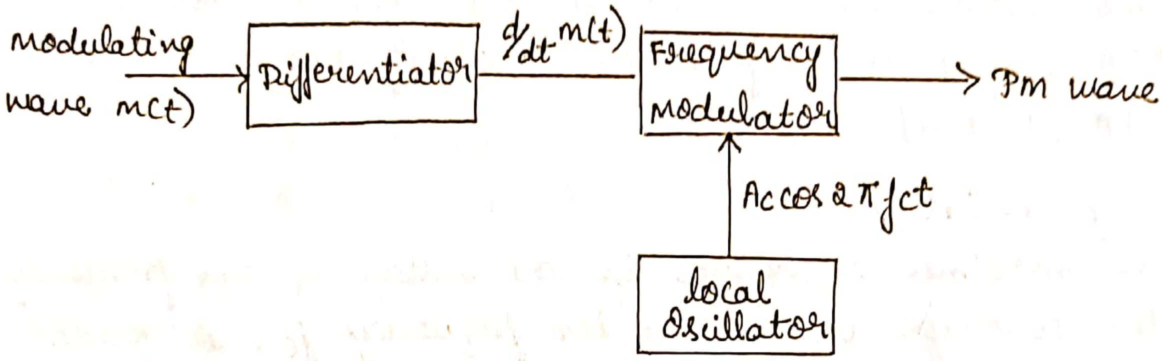
Fig: Generation of FM from phase modulator

FM wave can be generated by first integrating m(t) and then it is used as input for phase modulator results in FM wave at the output as shown in figure.

$$\left\langle s(t) = A_c \cos\left[2\pi f_c t + \underbrace{2\pi k_f}_{K_p} \int_0^t m(t)\, dt\right] \right\rangle$$

ii) <u>Generation of PM wave using FM [Frequency modulator]</u>

using frequency modulator, PM is generated by first differentiating modulating signal m(t) and then input to the frequency modulator as shown in below figure



W.K.T $\quad S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]$

Input to frequency modulator is $\frac{d}{dt} m(t)$

Thus $\quad = A_c \cos\left[2\pi f_c t + 2\pi k_f \cancel{\frac{d}{dt}} \int_0^t m(t)\, \cancel{dt}\right]$

$= A_c \cos\left[2\pi f_c t + 2\pi k_f\, m(t)\right]$

substituting $2\pi k_f = k_p$

$$\left\langle S_{PM}(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right] \right\rangle$$

<u>Single tone Frequency Modulation:</u>

W.K.T $\quad S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]$

Since single tone, consider $m(t) = A_m \cos 2\pi f_m t$

$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t A_m \cos 2\pi f_m t \cdot dt\right]$

$= A_c \cos\left[2\pi f_c t + 2\pi k_f A_m \int_0^t \cos 2\pi f_m t \cdot dt\right]$

$= A_c \cos\left[2\pi f_c t + \cancel{2\pi} k_f A_m \frac{\sin 2\pi f_m t}{\cancel{2\pi} f_m}\right]$

$S_{FM}(t) = A_c \cos[2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t]$ $[\because k_f A_m = \Delta f]$

$S_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$ $[\because \beta = \Delta f / f_m]$

where, Δf = frequency deviation
β = modulation index.

Modulation index:

It is the ratio of frequency deviation 'Δf' to the modulating frequency 'f_m'. $[\beta = \Delta f / f_m]$

In FM, the modulation index is very important, because it decides the bandwidth of the FM wave.

If modulation index is small, then FM is Narrow Band FM [NBFM], if β is large then resulting FM is wide Band FM [WBFM].

Frequency Deviation:

The maximum change in the instantaneous frequency from the average value carrier frequency 'f_c', is known as frequency deviation.

$$\langle \Delta f = k_f A_m \rangle$$

Transmission Bandwidth:

The FM wave consists infinite number of sidebands. Thus Bandwidth of a FM signal is infinite.

In practical, By carson's rule,

$$BW = 2(\Delta f + f_m)$$

Other forms

$BW = 2\Delta f(1 + \frac{f_m}{\Delta f})$ $BW = 2f_m(\Delta f / f_m + 1)$

$BW = 2\Delta f(1 + \frac{1}{\beta})$ $BW = 2f_m(\beta + 1)$

$[\because \beta = \Delta f / f_m]$

Scanned with CamScanner

Types of FM or classification of FM

Depending on the value of the modulation index 'β', FM wave is classified as follows.

1) Narrow-band FM (NBFM)
2) Wideband FM (WBFM)

Narrow Band FM [NBFM]:

If the modulation index is small value, i.e less than one radian then it is Narrow Band FM.

Generation of NBFM with block diagram and its spectrum

W.K.T Angle modulated wave is given by,

$$\underset{\text{modulated wave}}{S_{angle}(t)} = A_c \cos[2\pi f_c t + \phi] \longrightarrow (1)$$

using $\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$

$\because A = 2\pi f_c t, \quad B = \phi$

$$\underset{\text{modulated wave}}{S_{angle}(t)} = A_c[\cos 2\pi f_c t \cos\phi - \sin 2\pi f_c t \sin\phi] \longrightarrow (2)$$

In NBFM, β is small & ϕ is less than 1 radian, hence it possible to approximate

$$\left.\begin{aligned} \cos\phi &\simeq 1 \\ \sin\phi &\simeq \phi \end{aligned}\right\} \longrightarrow (3)$$

substitute Eq (3) in Eq (2), we get

$$\underset{NBFM}{s(t)} = A_c[\cos 2\pi f_c t \cdot 1 - \sin 2\pi f_c t \, \phi]$$

W.K.T = $S_{FM}(t) = A_c \cos[2\pi f_c t + 2\pi K_f \int_0^t m(t)\,dt] \longrightarrow (4)$

Compare Eq^n (4) and Eq^n (1),

$$\phi \text{ in FM} = 2\pi K_f \int_0^t m(t)\,dt$$

By substituting ϕ in $S_{NBFM}(t)$, we get

$$S_{NBFM}(t) = A_c \cos 2\pi f_c t - A_c \sin 2\pi f_c t \, \phi$$

$$S_{NBFM}(t) = A_c \cos 2\pi f_c t - A_c \sin 2\pi f_c t \cdot 2\pi K_f \int_0^t m(t)\,dt$$

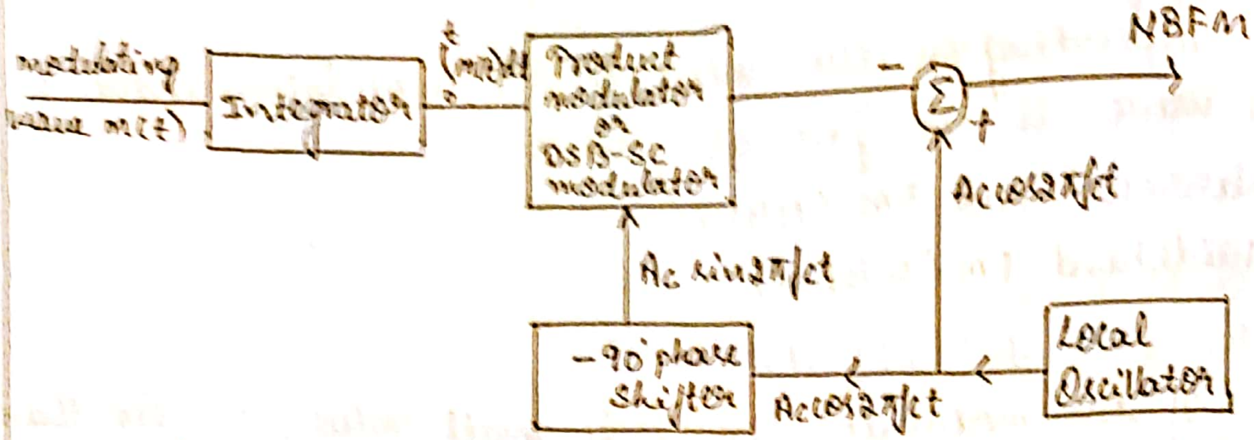


fig: Narrow Band phase modulator

Spectrum of NBFM:

we get,

$$S_{NBFM}(t) = A_c \cos 2\pi f_c t - 2\pi k_f A_c \sin 2\pi f_c t \int_0^t m(t)\, dt$$

Take $m(t) = A_m \cos 2\pi f_m t$

$$= A_c \cos 2\pi f_c t - 2\pi k_f A_c \sin 2\pi f_c t \int_0^t A_m \cos 2\pi f_m t \, dt$$

$$= A_c \cos 2\pi f_c t - 2\pi k_f A_m A_c \sin 2\pi f_c t \cdot \frac{\sin 2\pi f_m t}{2\pi f_m}$$

$$= A_c \cos 2\pi f_c t - \frac{k_f A_m A_c}{f_m} \sin 2\pi f_c t \, \sin 2\pi f_m t$$

$\because \Delta f = k_f A_m \qquad \beta = \frac{\Delta f}{f_m}$

$$= A_c \cos 2\pi f_c t - \frac{\Delta f}{f_m} A_c \sin 2\pi f_c t \, \sin 2\pi f_m t$$

$$= A_c \cos 2\pi f_c t - \beta A_c \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$

using $\sin A \cdot \sin B = -\frac{1}{2}[\cos(A+B) - \cos(A-B)]$

$$S_{NBFM}(t) = A_c \cos 2\pi f_c t + \frac{\beta A_c}{2}\left[\cos 2\pi (f_c + f_m)t - \cos 2\pi (f_c - f_m)t\right]$$

$$\left\{ S_{NBFM}(t) = A_c \cos 2\pi f_c t + \frac{\beta A_c}{2} \cos 2\pi (f_c + f_m)t - \frac{\beta A_c}{2} \cos 2\pi (f_c - f_m)t \right\}$$

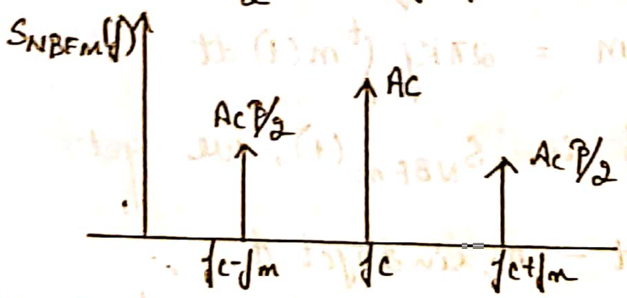


fig: Spectrum content of NBFM for single tone modulation

Note: * NBFM spectrum is same as AM conventional, it contains carrier signal, upper sideband (USB) & Lower Sideband (LSB).

* Transmission BW of NBFM is $2f_m$

Wideband FM:

If the modulation index is large value, then it is wideband FM.

Time domain expression for wideband or expression for the spectrum of FM wave:

consider single tone frequency modulation expression,

i.e $S_{STFM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$

using $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

here $A = 2\pi f_c t$, $B = \beta \sin 2\pi f_m t$

$$S_{STFM}(t) = A_c [\cos 2\pi f_c t \cdot \cos(\beta \sin 2\pi f_m t) - \sin 2\pi f_c t \cdot \sin(\beta \sin 2\pi f_m t)]$$

W.K.T $s(t) = s_I(t)\cos 2\pi f_c t - s_Q(t) \sin 2\pi f_c t$

from above Equ $s_I(t) = \cos(\beta \sin 2\pi f_m t)$

$s_Q(t) = \sin(\beta \sin 2\pi f_m t)$

from complex envelope, we have $\langle \tilde{s}(t) = s_I(t) + j s_Q(t) \rangle$

$$\tilde{s}(t) = A_c [\cos(\beta \sin 2\pi f_m t) + j \sin(\beta \sin 2\pi f_m t)]$$

W.K.T $= e^{j\theta} = \cos\theta + j \sin\theta$

$$\tilde{s}(t) = A_c e^{j(\beta \sin 2\pi f_m t)}$$

$\because \theta = \beta \sin 2\pi f_m t$.

Expanding using fourier series, in terms of Bessel function $J_n(\beta)$, $\tilde{s}(t)$ becomes

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}$$

Time domain expression for WBFM is

$$S_{WBFM}(t) = \mathrm{Re}[\tilde{s}(t) e^{j2\pi f_c t}]$$

$$= \mathrm{Re}\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} e^{j2\pi f_c t}\right]$$

$$= \mathrm{Re}\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi (f_c + n f_m)t}\right]$$

$(e^{j\theta})$

$$S_{WBFM}(t) = \mathrm{Re}\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) [\cos 2\pi (f_c + n f_m)t + j \sin 2\pi (f_c + n f_m)t]\right]$$

Taking only the real terms, we get

$$S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi (f_c + nf_m) t$$

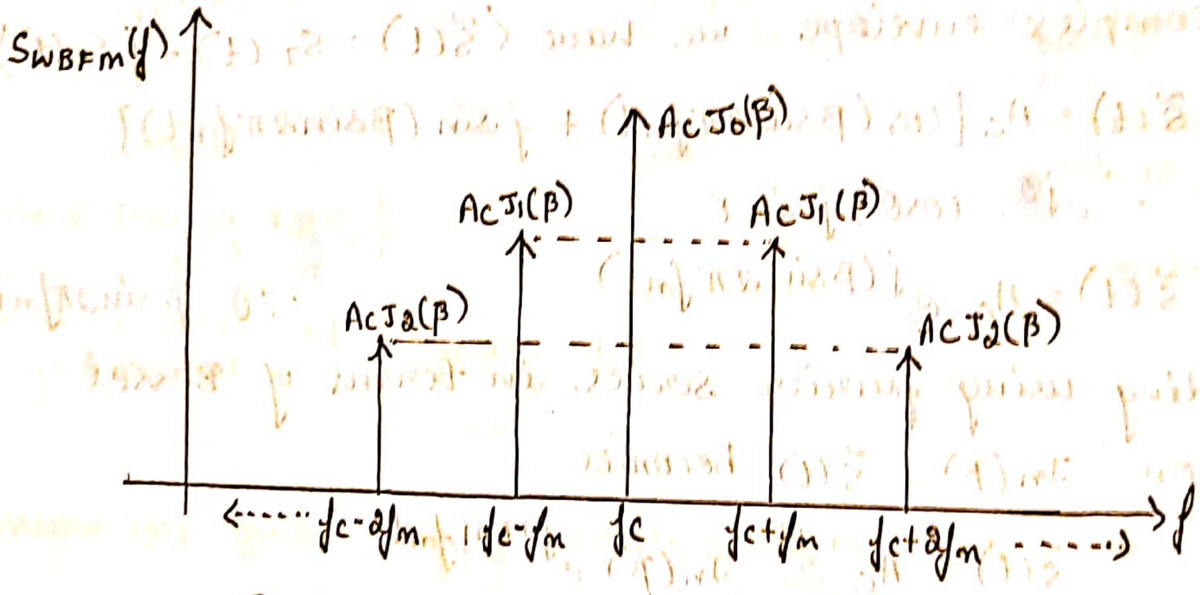
'n' lies between $-\infty, \infty$

i.e, $n = 0, +1, -1, +2, -2 \ldots \ldots +\infty, -\infty$

$$S_{WBFM}(t) = A_c [J_0(\beta) \cos 2\pi f_c t + J_1(\beta) \cos 2\pi (f_c + f_m) t + J_{-1}(\beta) \cos 2\pi (f_c - f_m) t + J_2(\beta) \cos 2\pi (f_c + 2f_m) t + J_2(\beta) \cos 2\pi (f_c - 2f_m) t + \ldots \ldots]$$

<u>Note</u>:

$$J_n(\beta) = \begin{cases} J_n(\beta) & \text{for } n = \text{even} \\ -J_n(\beta) & \text{for } n = \text{odd} \end{cases}$$



<u>Spectrum of WBFM</u>

Thus WBFM consists of carrier signal and infinite number of sidebands.

Differences between NBFM and WBFM

Parameters	NBFM	WBFM
* Modulation Index (β)	$\beta << 1$	$\beta >> 1$
* Spectrum	The spectrum of NBFM is same as that of AM (Contains 2 sidebands & carrier)	The spectrum of WBFM differs from AM. (Contains carrier & infinite number of sidebands)
* Bandwidth	Small	large
* Noise suppression	Poor	better
* Range of modulating frequency	30 Hz to 3 KHz.	30 Hz to 15 KHz
* Maximum Deviation $[\Delta f]_{max}$	5 KHz	75 KHz
* Transmission Quality	Low	High
* Applications	used in speech transmission Ex: FM mobile commn.	used for high quality music transmission. Ex: Entertainment broadcasting.

Generation of FM waves:

There are two basic methods of generating FM waves, namely,

1) Indirect method or Indirect FM:

In this method, a NBFM wave is generated, frequency multipliers are then used to increase the frequency deviation which results in wideband - FM (WBFM).

2) Direct FM or Direct method

In direct FM, the carrier frequency 'f_c' is directly varied in accordance with the amplitude of the modulating signal. Direct FM is not feasible, practically as it involves maintaining high frequency stability of the carrier with adequate frequency deviation.

Indirect FM:

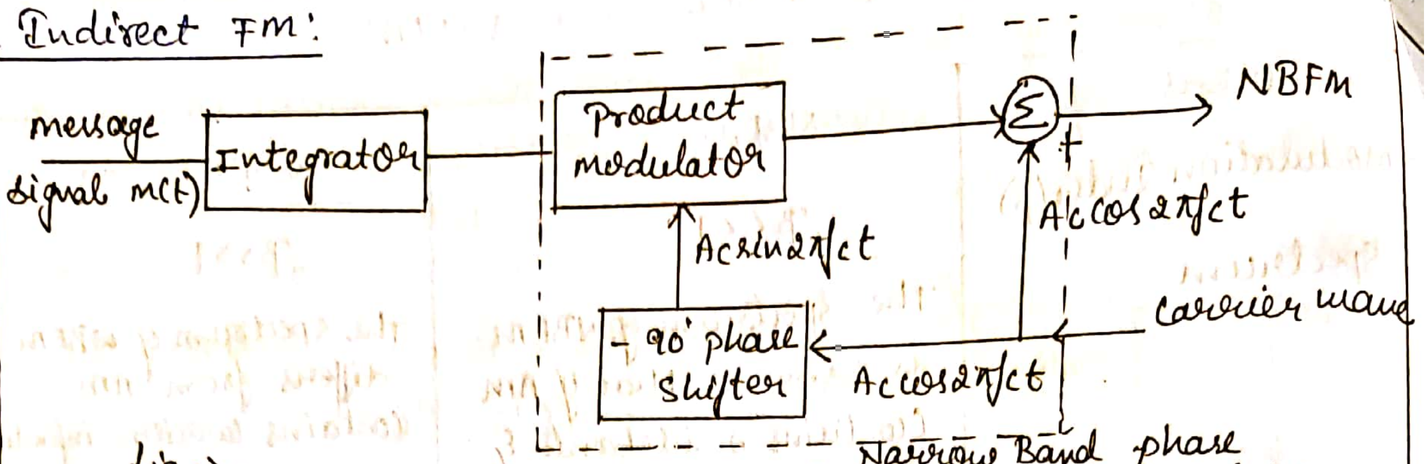


fig (a): NB frequency modulator

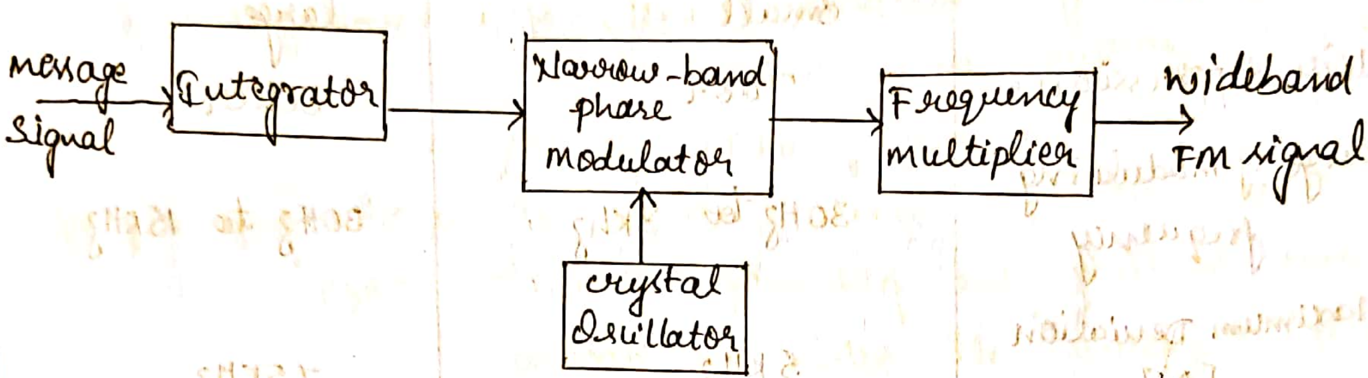
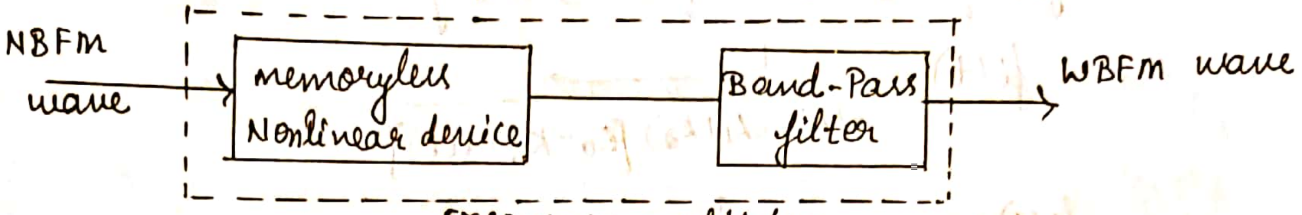


fig (b): wide-band frequency modulator.

* In indirect method, the message signal m(t) is first passed through an integrator before applying it to the phase modulator as shown in fig (a).
* The carrier signal is generated by using crystal Oscillator because it provides very high frequency stability.
* The operation of indirect method is divided into two parts as follows:
  - Generate a NBFM using a phase modulator
  - using the frequency multipliers & mixer to obtain the required values of frequency deviation and modulation index (i.e WBFM).
* In order to minimize the distortion in the phase modulator, the maximum phase deviation or modulation index 'β' is kept small there by resulting in a NBFM signal.

Generation of WBFM:

* The output of the Narrow band phase modulator is then multiplied by a frequency multiplier, providing the desired WBFM wave as shown in below figure



Frequency multiplier

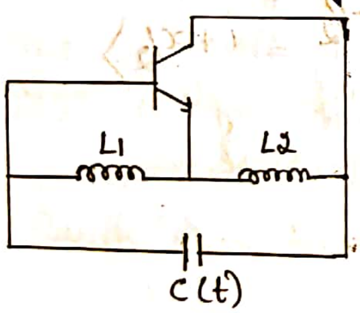
* A frequency multiplier consists of a memoryless non-linear device followed by a BPF.

The BPF has two functions to perform:

- To pass the FM wave centered at carrier frequency nf_1 and having the frequency deviation $n\Delta f_1$.
- To suppress all other FM spectra.

* The output of the frequency multiplier produces the desired wide-Band FM wave.

Direct method for FM Generation:



In the direct method of FM generation, the instantaneous frequency of the carrier wave is varied directly in accordance with the message signal by means of a device known as voltage-controlled Oscillator (VCO).

As an example, consider Hartley Oscillator as shown in fig. The capacitive component of the frequency determining network consists of a fixed capacitor shunted by a voltage variable capacitor, commonly known as varactor or varicap. Thus the resultant capacitance is $C(t)$.

The frequency of Oscillation of the Hartley Oscillator is given by

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C(t)}} \longrightarrow (1)$$

where, $C(t)$ is total capacitance of fixed & variable voltage Capacitor
L_1 & L_2 are two inductances in frequency determining network.

$$C(t) = C_0 - K_c\, m(t) \longrightarrow ②$$

Where. $C_0 \rightarrow$ capacitance value in the absence of modulation.
$K_c \rightarrow$ Variable capacitor's Sensitivity.

Substituting Eqⁿ (2) in Eqⁿ (1), we get

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\left[C_0 - K_c\, m(t)\right]}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\, C_0\left[1 - \frac{K_c}{C_0} m(t)\right]}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}} \cdot \frac{1}{\sqrt{\left[1 - \frac{K_c}{C_0} m(t)\right]}}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}} \quad \frac{1}{\left[1 - \frac{K_c}{C_0} m(t)\right]^{1/2}}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}} \quad \left[1 - \frac{K_c}{C_0} m(t)\right]^{-1/2}$$

$$f_i(t) = f_0\left[1 - \frac{K_c}{C_0} m(t)\right]^{-1/2}$$

$$\left\langle \text{Binomial theorem} \rightarrow [1-x]^{-1/2} = 1 + x/2 \right\rangle$$

$$f_i(t) = f_0\left[1 + \frac{K_c}{2C_0} m(t)\right]$$

$$f_i(t) = f_0 + \frac{K_c f_0}{2C_0} m(t)$$

$$\left\langle f_i(t) = f_0 + K_f\, m(t) \right\rangle$$

where, K_f = frequency sensitivity $= \frac{K_c f_0}{2C_0}$

Thus $f_i(t) = f_0 + K_f m(t)$ represents standard expression for instantaneous frequency of FM wave.

The major disadvantage is that there is no frequency stability.

Stability can be achieved by using feedback system as shown in figure.

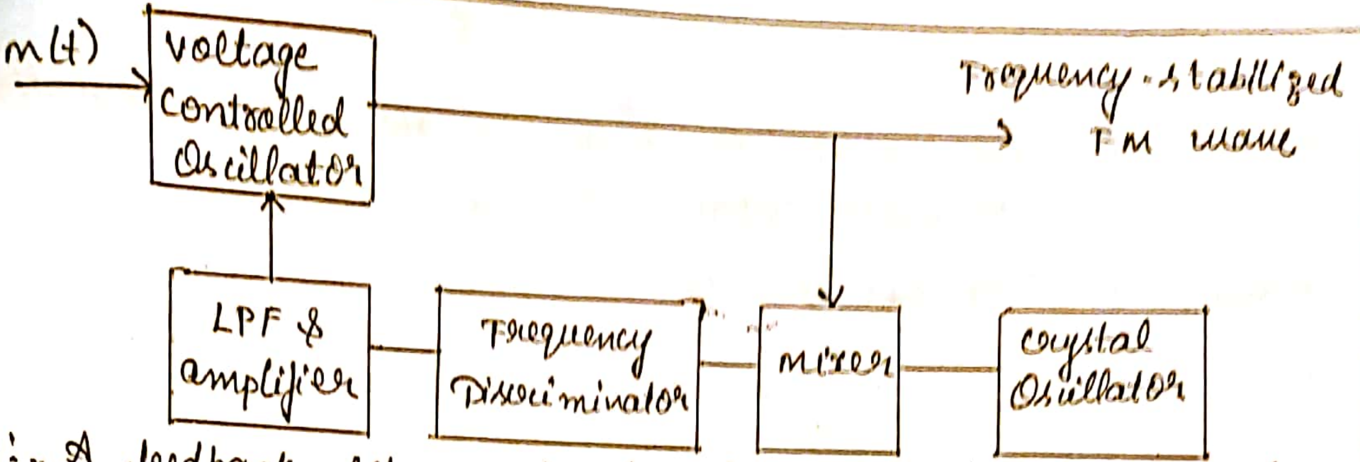


fig:- A feedback scheme for the frequency stabilization of a frequency modulator.

* Output of the FM generator is applied to a mixer together with the output of a crystal oscillator and the difference frequency term is extracted by mixer. Mixer O/P is next applied to a frequency discriminator and then low-pass filtered.

* When the FM transmitter has exactly the correct carrier frequency, the low-pass filter output is zero. However, deviations of the transmitter carrier frequency from its assigned value will cause the frequency discriminator-filter combination to develop a dc output voltage.

* The DC voltage, after amplification it is applied to the voltage controlled oscillator of the FM transmitter in such a way to modify the frequency of the oscillator.

Scanned with CamScanner

De-modulation of FM waves:-

→ Frequency de-modulation is the Process of recovering the original modulating wave from the frequency modulated wave.

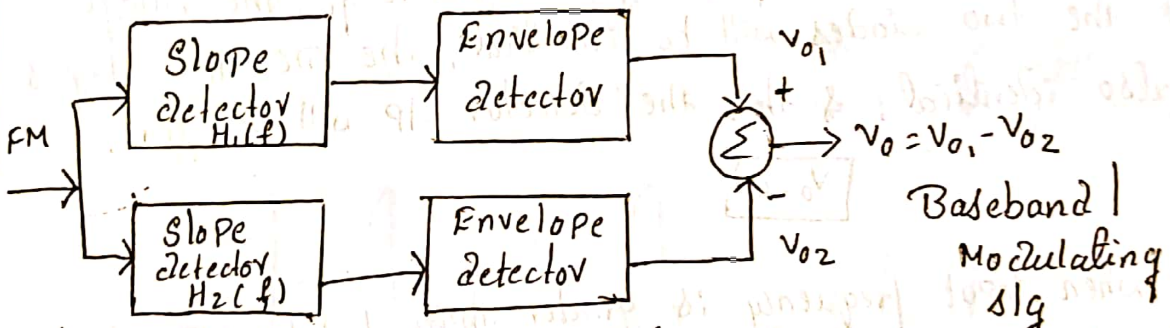
→ The FM-demodulators are classified into:-

i) Direct Method:- Eg:- Frequency discriminators, Zero crossing detectors.

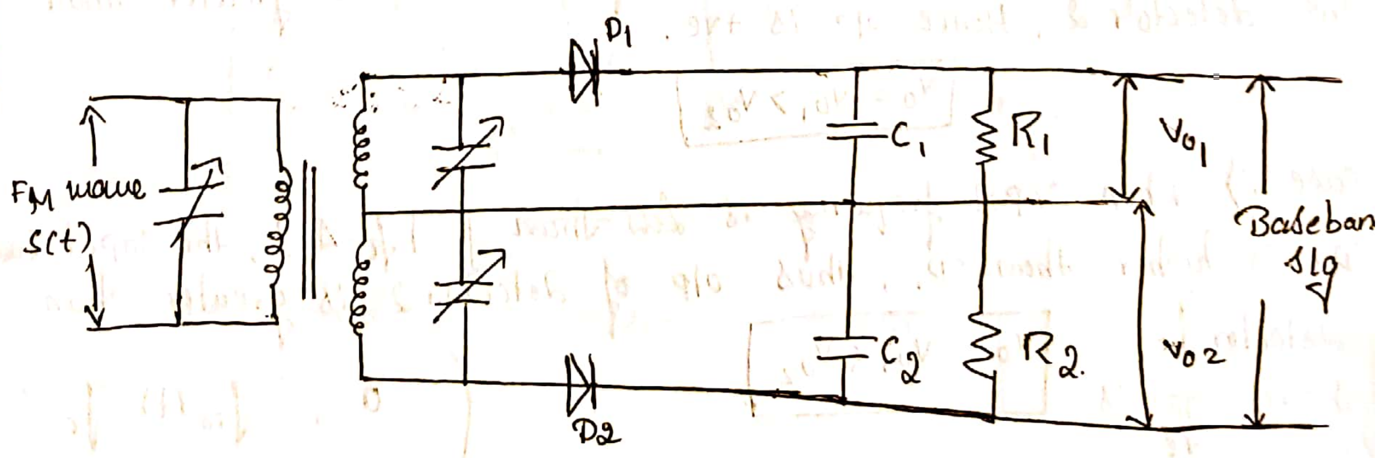
ii) Indirect Method:- Eg:- Phase-locked loop

Balanced Frequency discriminator | Balanced Slope detector:-

→ The model of the balanced discriminator as a Pair of slope circuits with their complex transfer functions, followed by envelope detector & a summer as shown in the fig(a)



fig(a): Idealized model of balanced frequency Discriminator



→ There are 2 tuned circuits

i) The Primary winding is tuned to frequency f_c

ii) Secondary winding is divided into 2 Parts.

→ The upper tuned circuit is tuned above f_c i,e $f_c + \Delta f$

→ The lower tuned circuit is tuned below f_c i,e $f_c - \Delta f$

→ R_1C_1 & R_2C_2 are the filter circuits, V_{o_1} & V_{o_2} are the o/p voltages of the two slope detectors.

→ The final output voltage V_o is obtained by taking the difference of the individual o/p voltages V_{o_1} & V_{o_2}.

$$\boxed{\text{i,e } V_o = V_{o_1} - V_{o_2}}$$

Case i) :- When the input frequency is equal to f_c, the voltage applied to the two diodes will be identical, the DC o/p voltages will be also identical, & thus the detector o/p will be zero.

$$\boxed{V_o = 0}$$

Case ii) :- When input frequency is greater than f_c [$f_c + \Delta f$], the input to D_1 is higher than D_2, thus o/p of detector 1 is greater than the detector 2, Hence o/p is +ve.

$$\boxed{V_o = V_{o_1} > V_{o_2}}$$

Case iii) When input frequency is less than f_c [$f_c - \Delta f$], the input to D_2 is higher than D_1, thus o/p of detector-2 is greater than detector-1.

$$\boxed{V_o = V_{o_1} < V_{o_2}}$$

Hence o/p is -ve.

$$\therefore V_o = \begin{cases} 0, & f_{in}(t) = f_c \\ +ve, & f_i(t) > f_c \\ -ve, & f_i(t) < f_c \end{cases}$$

Advantages :-

→ This circuit is more efficient than simple slope detector

→ It has better linearity than the simple slope detector

Disadvantages :-

→ It is difficult to tune since the 3 tuned circuits are to be tuned at different frequencies i,e f_c, $f_c+\Delta f$, $f_c-\Delta f$.

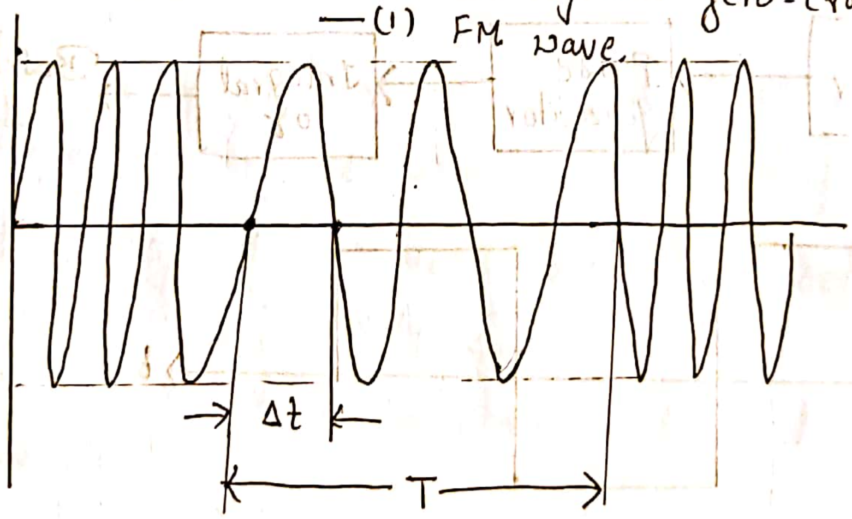
Zero-Crossing Detector :-

→ It is a frequency counter, which measures the instantaneous frequency by the number of zero-crossings. Then the rate of zero-crossings indicate the instantaneous frequency of the signal.

→ Zero-crossing detector mainly operates on the principle that the instantaneous frequency of an FM wave is given by,

$$\boxed{f_i \simeq \frac{1}{2\Delta t}} \quad \text{—(1)}$$

Δt; is the time difference b/n the adjacent zero-crossing of the FM wave.



→ The time interval 'T' is chosen in accordance with the following 2 conditions.

i) The interval 'T' is small compared to the reciprocal of the message bandwidth 'w' ($1/w$)

ii) The Interval 'T' is large compared to the reciprocal of the carrier frequency 'f_c' of the FM wave i,e $(1/f_c)$

→ Let n_0 denote the no: of zero-crossings inside the interval T. i,e $T = n_0 \Delta t$

$$\Delta t = T/n_0 \quad \text{--- (2)}$$

Then, Instantaneous $freq^n$ is given by,

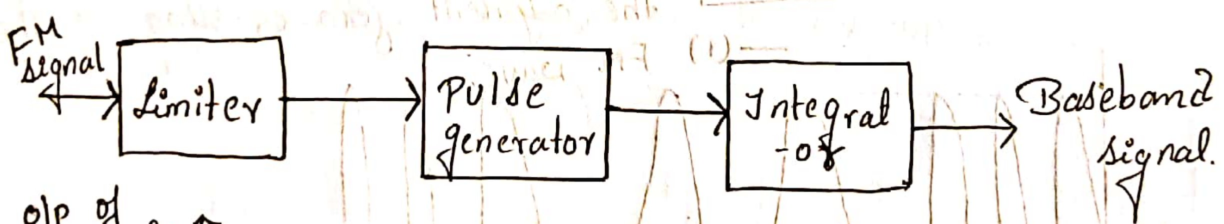
$$f_i = \frac{1}{2\Delta t}$$

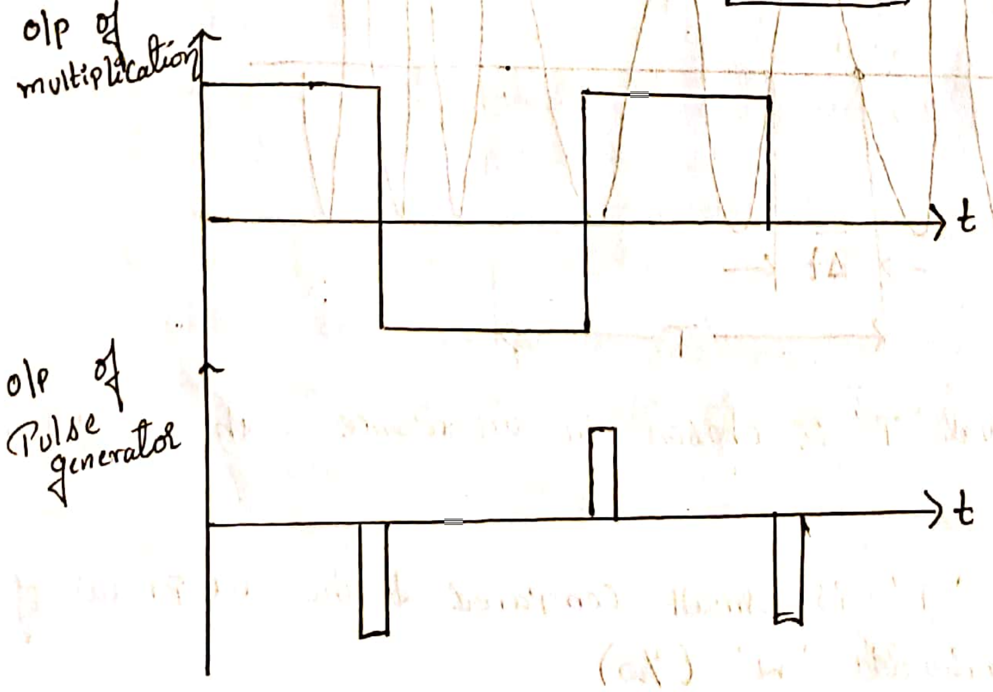
Now, substitute Eq^n (2) in (1)

$$f_i = \frac{1}{2(T/n_0)}$$

$$\boxed{f_i = \frac{n_0}{2T}}$$

Thus m(t) can be re-covered by counting n_0.





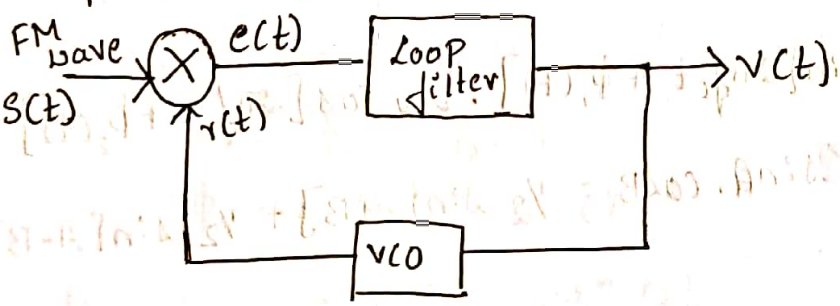
→ Limiter Produces a square wave version of the input FM wave.

→ The Pulse generator Produces a short pulses at the positive-going as well as negative-going Edges of the limiter o/p.

→ finally, integrator averages all the short pulses over an interval T, thus o/p reproduces the original modulating signal.

Phase-Locked Loop [PLL]:-

→ PLL is a -ve feedback system that consists of 3-major-components i) A multiplier
ii) A Loop filter
iii) voltage controlled oscillator [VCO], connected in the form of feedback loop as shown in the fig



→ Initially assume that VCO is adjusted, so that when the control voltage is zero, 2 conditions are satisfied

i) The frequency of the VCO, is Precisely set at the unmodulated carrier frequency 'f_c'

ii) The VCO o/p has a 90° Phase-shift w.r.t un-modulated-carrier wave.

→ Suppose that the i/p sig applied to the PLL is an FM wave defined by,

$$S_{FM}(t) = A_c \sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

$$S_{FM}(t) = A_c \sin\left[2\pi f_c t + \phi_1(t)\right] \quad \text{——(1)}$$

$\phi_1(t) = 2\pi k_f \int_0^t m(t).dt$; $k_f \rightarrow$ freq^n sensitivity

Let the VCO o/p be defined as

$$r(t) = A_v \cos\left[2\pi f_c t + 2\pi k_v \int_0^t v(t)\,.dt\right]$$

$$r(t) = A_v \cos\left[2\pi f_c t + \phi_2(t)\right] \quad \text{——} \quad (2)$$

$\phi_2(t) = 2\pi k_v \int_0^t v(t).\,dt$; $k_v \rightarrow$ freq^n sensitivity constant of the VCO

→ The o/p of the multiplier is,

$$e(t) = k_m s(t).\ r(t) \quad \text{——} \quad (3)$$ k_m = multiplier gain

substitute Eq^n (1) & (2) in Eq^n (3),

$$e(t) = k_m\ A_c \sin\left[2\pi f_c t + \phi_1(t)\right].\ A_v \cos\left[2\pi f_c t + \phi_2(t)\right]$$

$$\left\langle \sin A.\cos B = \tfrac{1}{2}\sin[A+B] + \tfrac{1}{2}\sin[A-B] \right\rangle$$

$$e(t) = \frac{k_m A_c A_v}{2}\left[\sin\left[2\pi f_c t + \phi_1(t) + 2\pi f_c t + \phi_2(t)\right] + \sin\left[\cancel{2\pi f_c t} + \phi_1(t) - \cancel{2\pi f_c t} - \phi_2(t)\right]\right]$$

$$e(t) = \frac{k_m A_c A_v}{2}\left[\sin\left[4\pi f_c t + \phi_1(t) + \phi_2(t)\right] + \sin\left[\phi_1(t) - \phi_2(t)\right]\right]$$

$$e(t) = \frac{k_m A_c A_v}{2}\sin\left[4\pi f_c t + \phi_1(t) + \phi_2(t)\right] + \frac{k_m A_c A_v}{2}\sin\left[\phi_1(t) - \phi_2(t)\right] \quad \text{——} \quad (4)$$

→ The o/p of the multiplier has 2 components

i) Highest freq^n. components, $\frac{k_m A_c A_v}{2}\sin\left[4\pi f_c t + \phi_1(t) + \phi_2(t)\right]$

ii) Lowest freq^n, components, $\frac{k_m A_c A_v}{2}\sin\left[\phi_1(t) - \phi_2(t)\right]$

highest frequency component is Eliminated by the LPF

$$e(t) = \frac{K_m A_c A_v}{2} \sin[\phi_1(t) - \phi_2(t)]$$

$$e(t) = \frac{K_m A_c A_v}{2} \sin[\phi_e(t)] \quad \text{—(5)}$$

$\phi_e(t)$ = Phase Error
$\phi_e = \phi_1(t) - \phi_2(t)$

Thus final o/p v(t) i,e

$$v(t) = e(t) * h(t)$$

$$v(t) = \frac{K_m A_c A_v}{2} \sin[\phi_e(t)] * h(t) \quad \text{—(6)}$$

→ To show o/p of PLL is scaled version of m(t):-

when the phase Error $\phi_e(t)$ is zero, the PLL is said to be in phase locked.

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

$$0 = \phi_1(t) - \phi_2(t)$$

$$\phi_1(t) = \phi_2(t) \quad \text{—(7)}$$

substitute Eq^n (1) & (2) in Eq^n (7).

$$\cancel{2\pi} k_f \int_0^t m(t).dt = \cancel{2\pi} k_v \int_0^t v(t).dt \quad \text{—(8)}$$

Differentiating above Eq^n (8) w.r.t to 't' on b.s

$$\frac{d}{dt} k_f \int_0^t m(t).dt = \frac{d}{dt} K_v \int_0^t v(t).dt$$

$$k_f m(t) = K_v v(t)$$

$$\boxed{v(t) = \frac{k_f}{K_v} m(t)} \quad \text{—(9)}$$

Thus v(t), o/p of PLL is scaled version of m(t) or

$$v(t) \propto m(t)$$

→ Comparison of AM & FM systems :-

Parameter	AM	FM
Definition	Amplitude of the carrier signal is changed w.r.t modulating signal	Frequency of the carrier signal is changed w.r.t modulating signal
Spectrum	It has only 2 sidebands	It has 'n' no. of sidebands
Modulation Index 'β' [M.I]	$\mu < 1$	$\beta > 1$
M.I formula	$\mu = \frac{A_m}{A_c}$	$\beta = \frac{\Delta f}{f_m}$
B.W	$2f_m$	$2(\Delta f + f_m)$
Transmitted Power, P_T	$P_T = P_c\left[1 + \frac{\mu^2}{2}\right]$	$P_T = \frac{A_c^2}{2R}$
Application	Long dist communication	Short dist communication

→ Comparision of FM & PM systems :-

Parameter	FM	PM
Instantaneous freq^n $f_i(t)$	$f_i(t) = f_c + k_f\, m(t)$ $f_i(t) \propto m(t)$	$f_i(t) = f_c + \frac{k_p}{2\pi} \frac{d\, m(t)}{dt}$ $f_i(t) \propto \frac{d}{dt} m(t)$
Noise Suppression	Better	Poor
SNR	Better than PM	It is inferior to that of FM
Application	FM broadcasting	Mobile systems.

o/p of the amplifier is denoted by the letter 'f_s'.

→ Mixer will Perform super heterodyne function, with which will Produce sum of 2 signals or differences of 2 signals.

→ one of the output is from RF Amplifier, other output is from Local oscillator which generates the carrier frequency which ranges from 1005 KHZ to 2105 KHZ. which is represented by f_c.

→ These 2 signals are given to the mixer, the o/p of the mixer is $f_c + f_s$ or $f_c - f_s$. Here we are considering $f_c - f_s$.

→ Because, only 455 KHZ freq^n signal should be applied to the IF-Amplifier, if we apply any frequency other than 455KHZ it will reject the signal. so if we subtract $f_c - f_s$ i.e ⟨1005KHZ - 550KHZ = 455KHZ.⟩ we get 455KHZ frequency slg.

→ IF Amplifier will amplify the $f_c - f_s$ signal, & it is given to Detector block.

→ Detector will Perform two operations;

i) <u>Rectification</u> :- Negative half cycle is eliminated, only Positive half cycle is Passed to filter.

Noise

* Introduction :-

→ Noise is a distrubance, an unwanted signals. Noise is a random Process in nature & interferes with desired signal.

→ Noise disturbs the Proper reception & reproduction of transmitted signals. Depending on the source which Produces noise, it is classified into, 2 types

i) External Noise:-

→ **Atmospheric Noise:-** which occurs due to Electrical distrubance such as lightining etc

→ **Extraterrestrial Noise:-** It is classified into 2-types

i) Solar Noise :- which causes due to sun

ii) Cosmic Noise :- which causes due to stars

→ **Industrial Noise :-** It is also called as man-made Noise, it is manily generated in auto-mobile & aircraft Industries.

2) Internal Noise:-

→ **Shot Noise:-**

→ The current in an electronic device, such as diode or transistor, under DC-condition is constant at every instant of time because of flow of electrons & holes.

→ The fluctuations in the no: of electrons results in Shot Noise.

→ In the Photodiode, electrons are emitted at random times τ_K when the light falls on the junction. Hence the current pulses generated will give this current

i.e $$x(t) = \sum_{K=-\infty}^{\infty} h(t-\tau_K)$$

where, $x(t)$ denotes the Photo current

$h(t-\tau_K)$ denotes the Pulse generated time $t = \tau_K$

→ **Thermal Noise** :-

→ The free Electrons with in an electrical conductor possess kinetic energy, when heat exchange takes place b/n the conductor & surroundings.

→ This motion of free electrons is randomized through collision due to imperfections in the conductor structure. Thus thermal noise is the electrical noise arising from the random motion of free electrons in a conductor.

→ The power spectral density of thermal noise produced by a resistor is given by,

$$S_{TN}(f) = \frac{2h|f|}{\exp\left[\frac{h|f|}{KT}\right] - 1}$$

we can use approximation,

$$\exp\left[\frac{h|f|}{KT}\right] = 1 + \frac{h|f|}{KT}$$

$$S_{TN}(f) = \frac{2h|f|}{1 + \frac{h|f|}{KT} - 1} = \frac{2\cancel{h|f|}}{\cancel{h|f|}} \cdot KT$$

$$\boxed{S_{TN}(f) = 2KT}$$

The mean square value of thermal noise vtg, measured across the terminal of the resistor is given by,

$$E_{TN}^2 = 2RB_N[S_{TN}(f)]$$

$$E_{TN}^2 = 2RB_N[2KT]$$

$$\boxed{E_{TN}^2 = 4KTB_N}$$

E_{TN} = root-mean square noise vtg
K = Boltzman's const = 1.38×10^{-23} J/K
T = Temp in kelvins
B_N = Noise B.W
R = Resistance

→ The root mean square value of the noise current is given as,

$$I_{TN}^2 = \frac{E_{TN}^2}{R^2} = \frac{4KTB_N \cancel{R}}{R^{\cancel{2}}}$$

$$\boxed{I_{TN}^2 = 4KTB_N G \text{ amp}^2} \quad \langle 1/R = G \rangle$$

WKT, $V = IR$

$I = \frac{V}{R}$

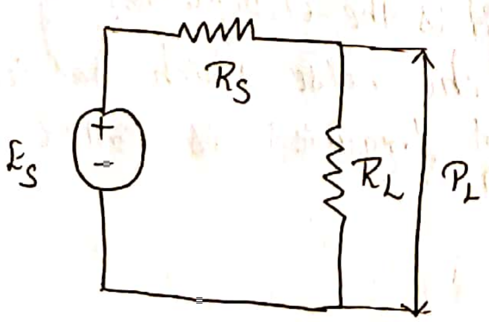
$I = \frac{E}{R}$

Squaring on b.s,

$\left\langle I^2 = \frac{E^2}{R^2} \right\rangle$

Note:-

i) Consider a voltage source having an internal emf of E_S & internal resistance R_S with load R_L. For maximum power is delivered when $R_S = R_L = R$



WKT, $P = \frac{V^2}{R}$ — (1)

$V_{rms} = \frac{V_m}{\sqrt{2}}$ — (2) squaring on b.s

$\left\langle V_{rms}^2 = \frac{V_m^2}{2} \right\rangle$

Assume, $V = V_{rms}$

$$P = \frac{V_{rms}^2}{R}$$

$$P = V_m^2/2R$$

$\langle$ Assume $V_m^2 = E_{TN}^2$ $\rangle$

$$P_{L\,max} = \frac{E_{TN}^2}{2R} = \frac{4KTRB_N}{2[R_L+R_S]} \quad \langle R_L = R_S = R \rangle$$

$$P_{L\,max} = \frac{E_{TN}^2}{2} = \frac{4KTRB_N}{2[2R]} = \frac{\cancel{4}KT\cancel{R}B_N}{\cancel{4}\cancel{R}}$$

$$\boxed{P_{L\,max} = KTB_N}$$

ii) Series Combination:-

$$\langle R_{eq} = R_1 + R_2 + \cdots + R_n \rangle$$

$$E_{TN}^2 = 4KTR_{eq}B_N$$

$$E_{TN}^2 = 4KTB_N[R_1 + R_2 + \cdots + R_n]$$

$$E_{TN}^2 = 4KTB_NR_1 + 4KTB_NR_2 + \cdots + 4KTB_NR_n$$

$$\boxed{E_{TN}^2 = E_{TN_1}^2 + E_{TN_2}^2 + \cdots + E_{TN_n}^2}$$

iii) Parallel Combination:-

$$E_{TN}^2 = 4KTR_{eq}B_N \quad \langle 1/R_{eq} = 1/R_1 + 1/R_2 + \cdots + 1/R_n \rangle$$

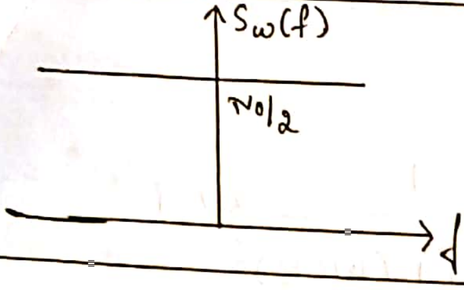
→ White Noise:-

→ White noise is not the noise source, it is the classification of noise, also known as constant noise. The noise which has constant noise power over all the range of frequencies is called white noise.

→ Spectral power density is given by,

$$S_w(f) = N_0/2$$

$$N_0 = KTe$$

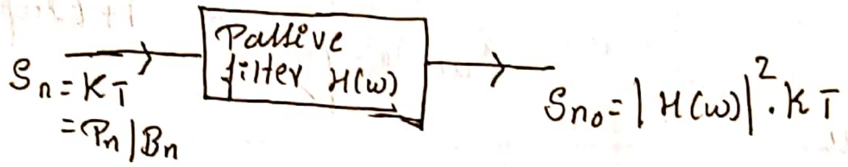


K = Boltzman's constant $= 1.38 \times 10^{-23}$ J/K

T_e = temp Equivalent Noise.

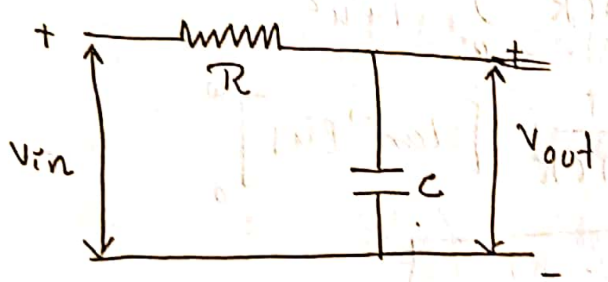
→ Noise Equivalent Bandwidth :-

→ Consider a Passive filter having voltage-ratio transfer funⁿ $H(\omega)$. Let the input noise spectrum density be $S_n = KT = \frac{P_{IN}}{B_N}$, where P_{IN} is noise power



The o/p noise spectrum density S_{no} for an input density of $S_n = KT$ is

$$S_{no} = |H(\omega)|^2 . KT \quad \text{—— (1)}$$



T.F ⇒ R = R

⇒ C = $1/cs$

Consider the Passive RC-low Pass filter shown in the fig,

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{1/cs}{R + 1/cs} = \frac{1}{1+sCR}$$

$\langle j\omega \to s \rangle$

$$H(j\omega) = \frac{1}{1+j\omega CR} \Rightarrow |H(j\omega)| = \frac{1}{|\underset{a}{1} + j\underset{b}{\omega CR}|}$$

$\langle |a+jb| = \sqrt{a^2+b^2}$

$$H(\omega) = \frac{1}{\sqrt{(1)^2 + (\omega CR)^2}}$$

$$\therefore S_{no} = |H(\omega)|^2 . kT$$

$$= \left[\frac{(1)^2}{\sqrt{1+(\omega CR)^2}}\right]^2 \times kT \Rightarrow \frac{1}{1+(\omega CR)^2} \times kT$$

The tot Noise Power at the o/p is obtained by integral-ing S_{no} over the entire freq^n spectrum for 0 to ∞.

$$P_{no} = \int_0^\infty S_{no} . df = \int_0^\infty \frac{kT}{1+(\omega CR)^2} . df = \int_0^\infty \frac{kT}{1+(2\pi f CR)^2} df$$

$$\langle \omega = 2\pi f \rangle$$

Let $2\pi f CR = u$

differentiation w.r.t f

$$2\pi CR = \frac{du}{df}$$

$$df = \frac{du}{2\pi CR}$$

$$P_{no} = \int_0^\infty \frac{1}{1+u^2} . \frac{du}{2\pi CR} \times kT$$

$$= \frac{kT}{2\pi CR} \int_0^\infty \frac{1}{1+u^2} . du$$

$$= \frac{kT}{2\pi CR} \left[\tan^{-1}[u]\right]_0^\infty$$

$$P_{no} = \frac{kT}{2\cancel{\pi} CR}\left[\cancel{\pi}/2\right] \Rightarrow \boxed{P_{no} = \frac{kT}{4CR}}$$

comparing with $P_{IN} = kTB_N$, we get

$$\frac{\cancel{kT}}{4RC} = \cancel{kT} B_N \qquad \boxed{B_N = \frac{1}{4RC}}$$

<u>Note</u>:- WKT, $E_{IN}^2 = 4kTRB_N$

$$= \cancel{4}kT\cancel{R} . \frac{1}{\cancel{4}\cancel{R}C}$$

$$\boxed{E_{IN}^2 = \frac{4kT}{C}}$$

Signal to Noise Ratio :- [SNR]

SNR is defined as the ratio of signal Power to the noise power. i,e SNR = $\frac{\text{Signal Power}}{\text{Noise Power}}$

$$SNR = \frac{P_S}{P_N} \quad \langle P = \frac{V^2}{R} \rangle$$

$$SNR = \left[\frac{V_s^2/R}{V_n^2/R}\right] \Rightarrow SNR = \frac{V_s^2}{V_n^2} \Rightarrow \left[\frac{V_s}{V_n}\right]^2$$

$$(SNR)_{db} = 10 \log\left[\frac{V_s}{V_n}\right]^2 \Rightarrow 20 \log\left[\frac{V_s}{V_n}\right]$$

Noise Factor :- [F]

The noise factor 'F' of an amplifier, or any other network is defined as

$$F = \frac{\text{available SNR Power at the input}}{\text{available SNR Power at the output}}$$

$$= \frac{(P_S/P_N)_i}{(P_S/P_N)_0} = \frac{P_{Si}/P_{Ni}}{P_{So}/P_{No}}$$

$$\therefore \quad F = \frac{P_{Si}}{P_{Ni}} \times \frac{P_{No}}{P_{So}}$$

Noise figure :- [N.F]

If the noise factor F is expressed in decibles it is known as "Noise figure".

$$\text{Noise figure} = 10 \log [F]$$

$$= 10 \log \left[\frac{P_{Si}/P_{Ni}}{P_{So}/P_{NO}} \right] \qquad \langle \log(m/n) = 10\log m - 10\log n \rangle$$

$$\text{Noise figure} = 10 \log_{10}\left[\frac{P_{Si}}{P_{Ni}}\right] - 10 \log_{10}\left[\frac{P_{So}}{P_{NO}}\right]$$

If $F = 1$, Noise figure $= 10 \log(1) = 0\,db$

Thus ideal value of noise fig. is 0 db.

Note :- i) Expression for o/p noise power in terms of noise factor F is P_{NO}

$$F = \frac{P_{Si}}{P_{Ni}} \times \frac{P_{NO}}{P_{So}}$$

$$P_{NO} = F \times \frac{P_{So}}{P_{Si}} \times P_{Ni} \qquad \langle P_{So}/P_{Si} = G \rangle$$

$$P_{NO} = G F P_{Ni} \Rightarrow \boxed{P_{NO} = G F K T B_N}$$

ii) Amplifier input Noise in terms of F is

$$P_{na} = P_{Ni} - P_{TN} \qquad \langle P_{TN} = K T B_N \rangle$$

$$P_{na} = F K T B_N - K T B_N$$

$$\boxed{P_{na} = (F-1) K T B_N}$$

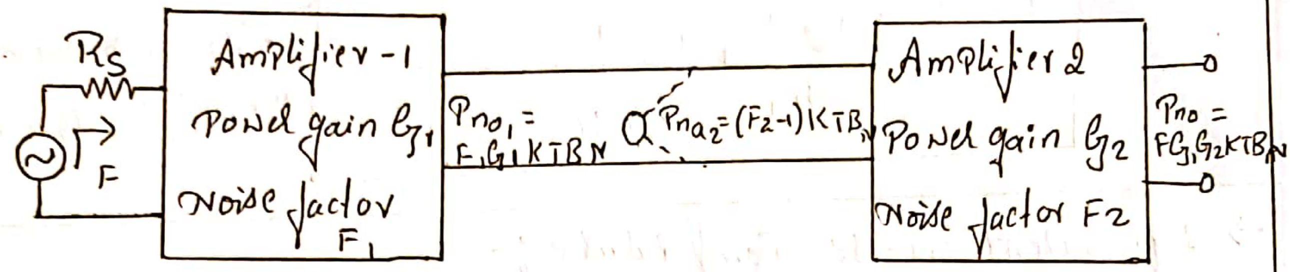
$$G = \frac{P_{NO}}{P_{Ni}} \Rightarrow P_{Ni} = \frac{P_{NO}}{G}$$

$$P_{Ni} = \frac{\cancel{G} F K T B_N}{\cancel{G}}$$

$$\langle P_{Ni} = F K T B_N \rangle$$

Cascade Connection of 2-port Networks:-

→ Consider 2-amplifier connected in cascade as shown in fig,



→ The available noise power at the o/p of first amplifier is $P_{no} = F_1 G_1 KTB_N$ & this is available to the second amplifier. The second amplifier has noise $[F_2 - 1] KTB_N$ of it's own at it's input of the second amplifier is,

$$P_{ni_2} = P_{no_1} + P_{na_2}$$

$$\boxed{P_{ni_2} = F_1 G_1 KTB_N + (F_2 - 1) KTB_N}$$

→ Considering 2^{nd} amplifier as a noiseless amplifier with amplifier gain G_2, we have

$$P_{no_2} = G_2 P_{ni_2}$$

$$\boxed{P_{no_2} = G_2 [F_1 G_1 KTB_N + (F_2 - 1) KTB_N]}$$

→ Overall gain of 2-amplifiers in cascade is $G = G_1 G_2$ & overall noise power $P_{no} = FG_1 G_2 KTB_N$, Equating Expressions for P_{no} & P_{no_2} we get,

$$FG_1 G_2 KTB_N = G_2 [F_1 G_1 KTB_N + (F_2 - 1) KTB_N]$$

$$F = \frac{\cancel{G_1} \cancel{G_2} F_1 \cancel{KTB_N}}{\cancel{G_1} \cancel{G_2} \cancel{KTB_N}} + \frac{(F_2 - 1) \cancel{KTB_N} \cancel{G_2}}{G_1 \cancel{G_2} \cancel{KTB_N}}$$

$$F = F_1 + \frac{F_2 - 1}{G_1}$$ < 2-cascade Amplifiers >

For 'N' no: of Amplifiers,

$$\boxed{F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots}$$

This is known as "Frii's" formula.

→ Equivalent Noise Temperature :-

Noise Power due to amplifier, having a noise factor F is $P_{na} = (F-1)KTB_N$

If T_e represents Equivalent noise temperature representing noise power, then

$$P_{na} = KT_eB_N.$$

Equating, $K T_e \cancel{B_N} = (F-1)\cancel{K}T\cancel{B_N}$

$$\boxed{T_e = [F-1]T} \quad \text{—— (1)}$$

using Frii's formula, Equivalent noise temperature of overall circuit having no: of Amplifiers connected in cascade, can be found as follows,

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cancel{+} \cdots$$

subtract 1 from both sides

$$F - 1 = F_1 - 1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots$$

If T_e is overall Equivalent noise temperature of the the cascade, while T_{e1}, T_{e2} ----- are corresponding values

.. each amplifier in cascade, then from Eq^n (1)

$$\frac{Te}{T} = \frac{Te_1}{T} + \frac{Te_2/T}{G_1} + \frac{Te_3/T}{G_1 G_2} + \cdots$$

$$\cancel{\frac{1}{T}}\,[Te] = \cancel{\frac{1}{T}}\,[Te_1 + Te_2/G_1 + Te_3/G_1 G_2] + \cdots$$

$$\boxed{Te = Te_1 + \frac{Te_2}{G_1} + \frac{Te_3}{G_1 G_2} + \cdots}$$

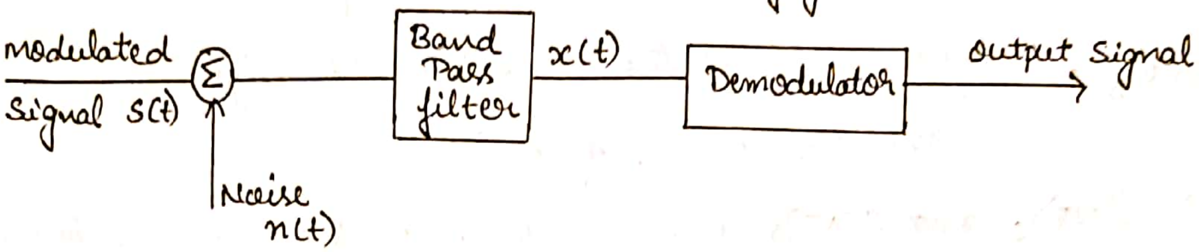
Module - 4

Noise in Analog Modulation

Noise performance of analog modulation system is evaluated by considering receiver model.

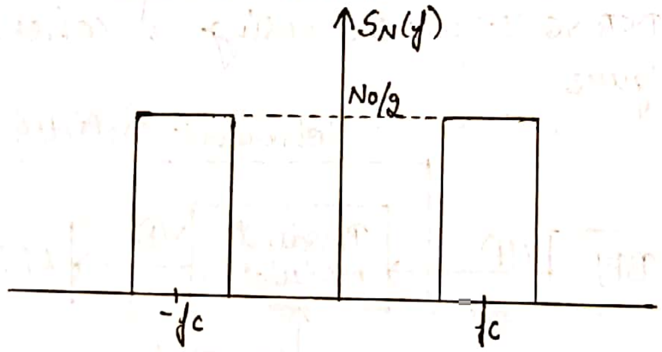
Receiver Model:

Receiver model is as shown in figure



In the above figure, s(t) is the modulated signal and n(t) is the noise signal. Signal n(t) is known as front end receiver noise. The receiver input signal is the sum of s(t) and n(t).

s(t) + n(t) passes through the Band pass filter, the bandwidth of BPF is equal to the transmission band width of the modulated signal. Demodulator used in the model depends on the type of modulation used.



Idealized characteristic of bandpass filtered noise

- Let $N_0/2$ is power spectral density [PSD] of Noise ~~n~~n(t) for both positive and negative frequency.
- N_0 is the average noise power per unit bandwidth.
- f_c is midBand frequency equal to center frequency.
- $f_c >> B_T$ so, filtered noise n(t) as a narrow band noise and it is defined in canonical form by,

$$\langle n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \rangle$$

$n_I(t)$ is inphase noise component & $n_Q(t)$ is quadrature noise co.

The filtered signal $x(t)$ at input of demodulator is

$$x(t) = s(t) + n(t)$$

- Channel signal to noise ratio is given by,

$$(SNR)_c = \frac{\text{average power of modulated signal}}{\text{Average power of noise in the message Bandwidth}}$$

- Output signal to noise ratio is given by,

$$(SNR)_o = \frac{\text{Average power of the demodulated signal}}{\text{Average power of noise}}$$

Both $(SNR)_c$ and $(SNR)_o$ are measured at the receiver side.

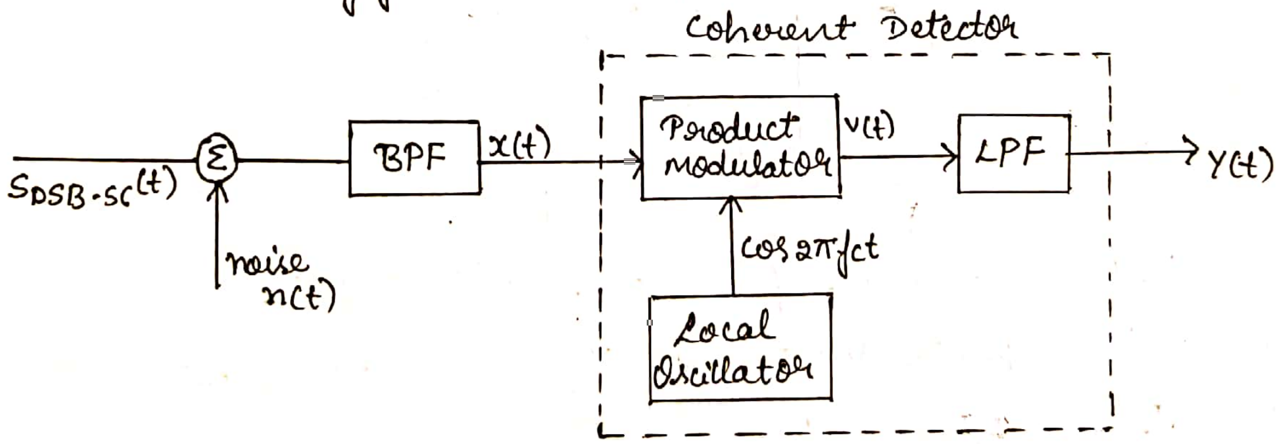
- Figure of merit for the receiver is given by,

$$\left\langle FOM = \frac{(SNR)_o}{(SNR)_c} \right\rangle$$

Higher the value of FOM, better the performance of the receiver. The value of FOM also depends on the type of modulation used.

<u>Noise in DSB-SC Receiver:</u>

The model of DSB-SC receiver using a coherent detector is shown in figure



- The filtered signal $x(t) = S_{DSB-SC}(t) + n(t)$ is applied to the coherent detector.
- In coherent detector, $x(t)$ is multiplied with a locally generated wave $\cos(2\pi f_c t)$ using product modulator.
- The output of product modulator $v(t)$ is filtered using LPF to get output.

W.K.T $\quad s_{DSB\text{-}SC}(t) = m(t)\, c(t)$

$= m(t) \cdot A_c \cos 2\pi f_c t$

$$FOM = \frac{(SNR)_o}{(SNR)_c}$$

Narrow-band noise $n(t)$ is given by

$n(t) = n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$

$$(SNR)_c = \frac{\text{Average power of modulated signal}}{\text{Average power of noise in message BW}}$$

Average power of modulated signal $= \dfrac{(A_c m(t))^2}{2} = \dfrac{A_c^2 m^2(t)}{2}$

$= \dfrac{A_c^2 P}{2} \qquad \because [m^2(t) = P]$

where P = average power of message.

Average power of noise in message Bandwidth $= N_0 W$

$$(SNR)_c = \frac{A_c^2 P/2}{N_0 W} = \frac{A_c^2 P}{2 N_0 W}$$

$$(SNR)_o = \frac{\text{Average power of Demodulated signal}}{\text{Average power of noise}}$$

From the DSB-SC receiver model

$x(t) = s_{DSB\text{-}SC}(t) + n(t)$

$= A_c m(t) \cos 2\pi f_c t + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$

Then,

$v(t) = x(t) \cos 2\pi f_c t$

$v(t) = A_c m(t) \cos^2 2\pi f_c t + n_I(t) \cos^2 2\pi f_c t - n_Q(t) \sin 2\pi f_c t \cos 2\pi f_c t$

$$v(t) = A_c m(t) \left[\frac{1 + \cos 4\pi f_c t}{2}\right] + n_I(t) \left[\frac{1 + \cos 4\pi f_c t}{2}\right] - n_Q(t) \frac{\sin 4\pi f_c t}{2}$$

$$v(t) = \frac{A_c m(t)}{2} + \frac{A_c m(t)}{2} \cos 4\pi f_c t + \frac{n_I(t)}{2} + \frac{n_I(t)}{2} \cos 4\pi f_c t - n_Q(t) \frac{\sin 4\pi f_c t}{2}$$

When $v(t)$ passes through LPF, output $y(t)$ is

$$y(t) = \frac{A_c m(t)}{2} + \frac{n_I(t)}{2}$$

Demodulated signal $= \frac{A_c m(t)}{2}$, Noise term $= \frac{n_I(t)}{2}$

$$(SNR)_o = \frac{\left(\frac{A_c m(t)}{2}\right)^2}{\frac{1}{2} N_0 W} = \frac{A_c^2 m^2(t)}{\cancel{4}\,2} \times \frac{\cancel{2}}{N_0 W} = \frac{A_c^2 P}{2 N_0 W}$$

$$(SNR)_o = \frac{A_c^2 P}{2 N_0 W}$$

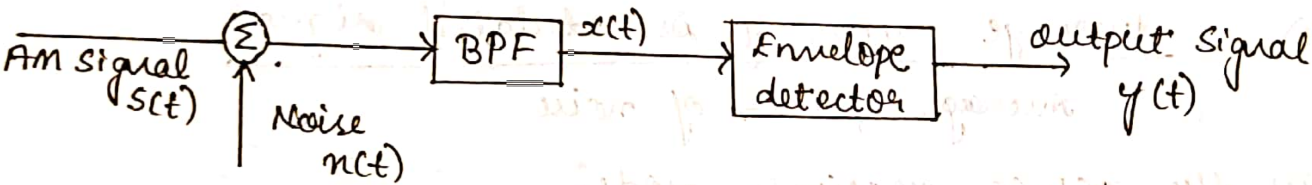
Then,

$$FOM = \frac{(SNR)_o}{(SNR)_c} = \frac{\cancel{A_c^2 P / 2 N_0 W}}{\cancel{A_c^2 P / 2 N_0 W}} = 1$$

$$\boxed{FOM = 1}$$

Noise in AM Receivers:

The model of AM receiver using envelop detector as demodulator is shown in figure.



The figure shows the model of an AM receiver, which used envelop detector for demodulation. The input signal s(t) and noise n(t) are added and given to BPF to make narrow band noise n(t). So filtered signal is $x(t) = s(t) + n(t)$. The Envelop detector produces the required AM demodulated signal y(t)

The i/p signal s(t) in an AM modulated wave is given by

$$S_{AM}(t) = A_c\,[1 + K_a m(t)] \cos(2\pi f_c t)$$

where, $A_c \cos 2\pi f_c t$ is the carrier wave

$m(t)$ is modulating signal

K_a is constant that determines the modulation Index.

Noise is given by,

$$n(t) = n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

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$$(SNR)_C = \frac{\text{Avg power of } s_{AM}(t)}{\text{Avg power of } n(t) \text{ in message BW}}$$

$$= \frac{\{A_c(1+k_a m(t))\}^2/2}{2N_0W} = \frac{A_c^2[1+k_a^2 m^2(t)]}{2N_0W}$$

$$(SNR)_C = \frac{A_c^2[1+k_a^2 P]}{2N_0W} \quad \because [m^2(t) = P]$$

To evaluate $(SNR)_o$

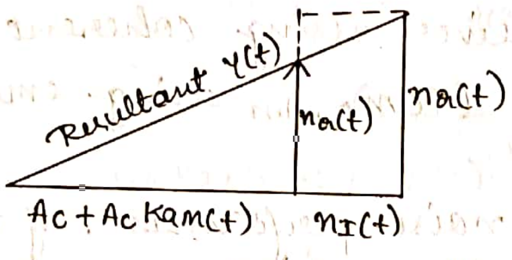
The signal $x(t)$ applied to the envelop detector in the receiver model.

$$x(t) = s_{AM}(t) + n(t)$$

$$x(t) = [A_c + A_c k_a m(t)]\cos 2\pi f_c t + n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t$$

$$x(t) = [A_c + A_c k_a m(t) + n_I(t)]\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t$$

The phasor diagram for signal $x(t)$ is



From this phasor diagram, the receiver output can be obtained as

$$y(t) = \text{Envelope of } x(t)$$

$$y(t) = \sqrt{[A_c + A_c k_a m(t) + n_I(t)]^2 + [n_Q(t)]^2}$$

when average power of carrier is large compared to average noise power. i.e $[A_c + A_c k_a m(t)]$ will be large compared with noise components $n_I(t)$ & $n_Q(t)$.

$y(t)$ is approximated as

$$y(t) \simeq A_c + A_c k_a m(t) + n_I(t)$$

The DC component A_c is removed by blocking capacitor, then

$$y(t) \simeq A_c k_a m(t) + n_I(t)$$

$$(SNR)_0 = \frac{[A_c K_a m(t)]^2}{2N_0W} = \frac{A_c^2 K_a^2 P}{2N_0W}$$

The above $(SNR)_{0,AM}$ is valid only if two conditions are satisfied.

1) The average noise power is small compared to the average carrier power at the envelope detector input.

2) The amplitude sensitivity K_a is less than or equal to 100%

$$FOM = \frac{(SNR)_0}{(SNR)_c} = \frac{A_c^2 K_a^2 P}{2WN_0} \Big/ \frac{A_c^2[1+K_a^2P]}{2WN_0}$$

$$= \frac{\cancel{A_c^2} K_a^2 P}{\cancel{2WN_0}} \times \frac{\cancel{2WN_0}}{\cancel{A_c^2}[1+K_a^2P]}$$

$$\boxed{FOM = \frac{K_a^2 P}{1+K_a^2 P}}$$

Note:

FOM of a DSB-SC receiver using coherent detection is always unity, whereas $(FOM)_{0,AM}$ using envelop detector is less than unity.

In other words, the noise performance of AM receiver is always inferior to that of DSB-SC receiver.

FOM of single tone AM Receiver:

For single tone, $m(t) = A_m \cos 2\pi f_m t$

$\therefore$ avg power of $m(t) = P = A_m^2/2$

W.K.T $(FOM)_{AM} = \dfrac{K_a^2 P}{1+K_a^2 P}$

$$(FOM)_{STAM} = \frac{K_a^2 A_m^2/2}{1+K_a^2 A_m^2/2} = \frac{\frac{K_a^2 A_m^2}{2}}{\frac{2+K_a^2 A_m^2}{2}}$$

$$(FOM)_{STAM} = \frac{(K_a A_m)^2}{2+(K_a A_m)^2}$$

$$\left\langle (FOM)_{STAM} = \frac{\mu^2}{2+\mu^2} \right\rangle \quad \because [\mu = K_a A_m \text{ in AM}]$$

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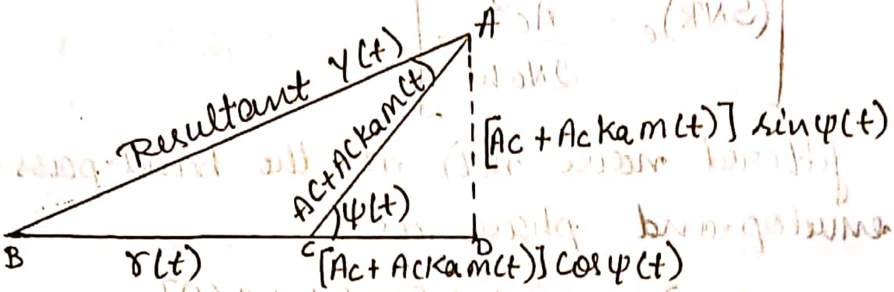
Threshold Effect:

When the carrier to noise ratio i.e $(CSNR)_C$ at the receiver input is small compared with unity, the noise term dominates and the performance of envelop detector changes.

In this case, $n(t)$ is represented in terms of its envelope $r(t)$ and phase $\psi(t)$ is

$$n(t) = r(t)\cos[2\pi f_c t + \psi(t)]$$

The corresponding phasor diagram is constructed with reference to $r(t)$ as shown, since noise dominates.



$$Y(t) = \sqrt{\{r(t) + [A_c + A_c k_a m(t)]\cos\psi(t)\}^2 + \{[A_c + A_c k_a m(t)]\sin\psi(t)\}^2}$$

here $r(t) \gg A_c$, so neglecting quadrature component of signal, we get O/P of detector is

$$Y(t) \simeq r(t) + [A_c + A_c k_a m(t)]\cos\psi(t)$$

$$Y(t) \simeq r(t) + A_c\cos\psi(t) + A_c k_a m(t)\cos\psi(t)$$

From the above expression, when the carrier to noise ratio is low, the detector output has no component strictly proportional to the message signal $m(t)$. The last term of the $y(t)$ i.e $A_c k_a m(t)\cos\psi(t)$ contains $m(t)$ multiplied by noise in the form of $\cos\psi(t)$.

Thus, the loss of information/message in an envelope detector that operates at a low carrier to noise ratio is referred to as the <u>threshold effect</u>.

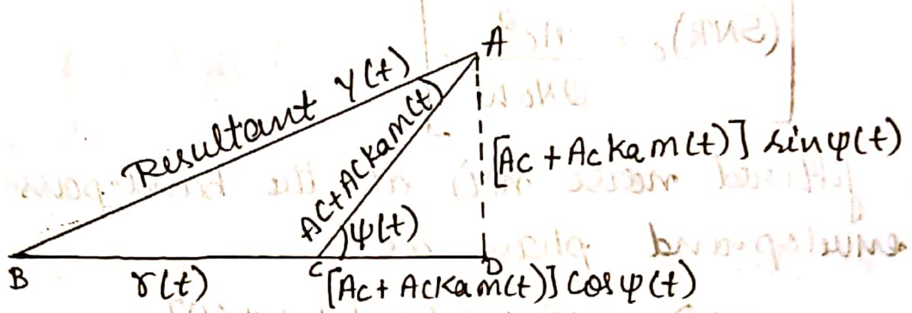
Threshold Effect:

When the carrier to noise ratio i.e $(SNR)_C$ at the receiver input is small compared with unity, the noise term dominates and the performance of envelop detector changes.

In this case, $n(t)$ is represented in terms of its envelope $r(t)$ and phase $\psi(t)$ is

$$n(t) = r(t) \cos[2\pi f_c t + \psi(t)]$$

The corresponding phasor diagram is constructed with reference to $r(t)$ as shown, since noise dominates.



$$Y(t) = \sqrt{\{r(t) + [A_c + A_c k_a m(t)]\cos\psi(t)\}^2 + \{[A_c + A_c k_a m(t)]\sin\psi(t)\}^2}$$

here $r(t) >> A_c$, so neglecting quadrature component of signal we get O/P of detector is

$$y(t) \simeq r(t) + [A_c + A_c k_a m(t)] \cos\psi(t)$$

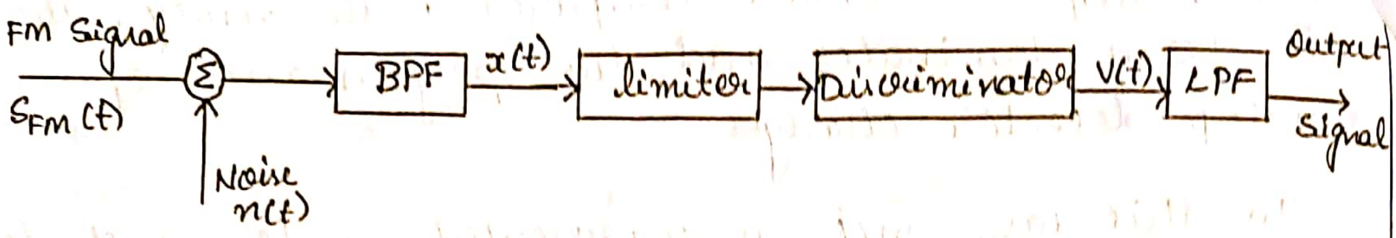
$$y(t) \simeq r(t) + A_c \cos\psi(t) + A_c k_a m(t) \cos\psi(t)$$

From the above expression, when the carrier to noise ratio is low, the detector output has no component strictly proportional to the message signal $m(t)$. The last term of the $y(t)$ i.e $A_c k_a m(t)\cos\psi(t)$ contains $m(t)$ multiplied by noise in the form of $\cos\psi(t)$.

Thus, the loss of information/message in an envelope detector that operates at a low carrier to noise ratio is referred to as the <u>threshold effect</u>.

Noise in FM Receiver:

FM receiver model is shown in figure



The incoming FM signal $S_{FM}(t)$ is given by

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

$$S_{FM}(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

Where $\phi(t) = 2\pi k_f \int_0^t m(t)\cdot dt$ $\qquad FOM = \frac{(SNR)_o}{(SNR)_c}$

$$\left[(SNR)_c = \frac{A_c^2}{2N_0W}\right]$$

The filtered noise n(t) at the band-pass filter output in its envelop and phase as

$$n(t) = r(t) \cos[2\pi f_c t + \psi(t)]$$

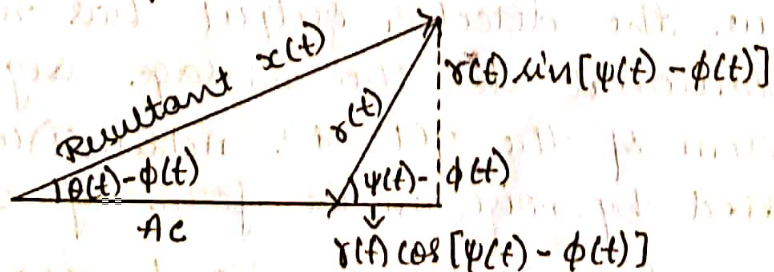
Where the envelope $r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$

and the phase is $\psi(t) = \tan^{-1}\left[\frac{n_Q(t)}{n_I(t)}\right]$

From the above figure, $x(t) = S_{FM}(t) + n(t)$

$$x(t) = A_c \cos[2\pi f_c t + \phi(t)] + r(t) \cos[2\pi f_c t + \psi(t)]$$

phasor diagram of x(t) is



$$\tan[\theta(t) - \phi(t)] = \frac{r(t)\sin[\psi(t) - \phi(t)]}{A_c + r(t)\cos[\psi(t) - \phi(t)]} \qquad \tan\theta = \frac{opp}{adj}$$

$$\psi(t) - \phi(t) = \tan^{-1}\left[\frac{r(t)\sin[\psi(t)-\phi(t)]}{A_c + r(t)\cos[\psi(t)-\phi(t)]}\right]$$

$$\theta(t) = \phi(t) + \tan^{-1}\left[\frac{r(t)\sin(\psi(t)-\phi(t))}{A_c + r(t)\cos(\psi(t)-\phi(t))}\right]$$

Assume signal to noise ratio at the discriminator input to be much larger than unity.

i.e $A_c >> r(t)$

$$A_c + r(t)\cos(\psi(t)-\phi(t)) \simeq A_c$$

$$\theta(t) = \phi(t) + \tan^{-1}\left[\frac{r(t)\sin[\psi(t)-\phi(t)]}{A_c}\right]$$

$$\theta(t) = \underbrace{2\pi k_f \int_0^t m(t)\,dt}_{\text{Signal term}} + \underbrace{\tan^{-1}\left[\frac{r(t)\sin[\psi(t)-\phi(t)]}{A_c}\right]}_{\text{Noise term}}$$

The discriminator output $v(t)$ is proportional to derivative of $\theta(t)$. i.e $\frac{d\theta(t)}{dt}$

i.e $$v(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

$$v(t) = \frac{1}{2\pi}\frac{d}{dt}\left\{2\pi k_f \int_0^t m(t)\,dt + \frac{r(t)\sin(\psi(t)-\phi(t))}{A_c}\right\}$$

$$v(t) = \frac{1}{2\pi} 2\pi k_f m(t) + \frac{1}{2\pi A_c}\frac{d}{dt}\left[r(t)\sin(\psi(t)-\phi(t))\right]$$

$$v(t) = k_f m(t) + n_d(t)$$

Thus output of the discriminator consists of original modulating signal $m(t)$ multiplied by scaling factor k_f, plus an additional noise component $n_d(t)$.

The Average power of output Noise is given by $\frac{2}{3}\frac{N_0 \omega^3}{A_c^2}$

Thus $[SNR]_o = \dfrac{[k_f m(t)]^2}{\frac{2}{3}\frac{N_o W^3}{A_c^2}}$

$$[SNR]_o = \frac{k_f^2 P 3 A_c^2}{2 N_o W^3}$$

$$FOM = \frac{[SNR]_o}{[SNR]_c} = \frac{3A_c^2 k_f^2 P}{2N_o W^3} \times \frac{2N_o W}{A_c^2}$$

$$\boxed{FOM = \frac{3k_f^2 P}{W^2}}$$

$$(FOM)_{FM} \propto \left(\frac{k_f}{W}\right)^2 \propto \left(\frac{\Delta f}{W}\right)^2 \propto \beta^2$$

W.K.T frequency deviation Δf is proportional to the frequency sensitivity k_f of the modulator.

FOM for single tone FM Receiver:

For single tone $m(t) = A_m \cos 2\pi f_m t$

Average power of $m(t) = P = A_m^2/2$

W.K.T $(FOM)_{FM} = \dfrac{3k_f^2 P}{W^2}$

$$= \frac{3k_f^2 A_m^2}{2W^2} = \frac{3(k_f A_m)^2}{2W^2} = \frac{3}{2}\left(\frac{\Delta f}{W}\right)^2$$

$$\boxed{(FOM)_{FM} = \frac{3}{2}\beta^2}$$

Capture Effect:

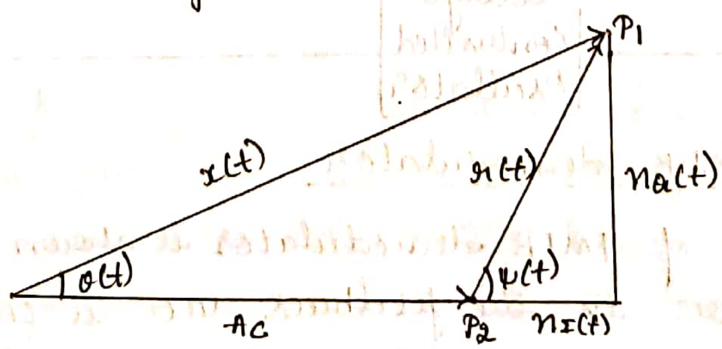
- In the frequency modulation, the signal can be affected by another frequency modulated signal whose frequency content is close to the carrier frequency of the desired FM wave.

The receiver may lock to interference signal and suppress the desired FM signal, when interference signal is stronger than the desired signal.

- When the strength of the desired signal and the interference signal are nearly equal, the receiver fluctuates back and forth between them, in this case receiver locks interference signal for some time and desired signal for the some time. This Phenomenon is known as the capture Effect.

FM Threshold Effect:

- The output signal-to-noise ratio of an FM signal indicated by the equation i.e $[SNR]_0 = \frac{3A_c^2 k_f^2 P}{2 N_0 W^3}$ is valid only if the carrier-to-noise ratio, measured at the discriminator input is high compared with unity.
- If the input noise power increases, the carrier-to-noise ratio decreases and receiver breaks.
- Initially, individual clicks are heard in the receiver output, and as the carrier-to-noise ratio decreases still further, the clicks rapidly merge into a crackling or sputtering sound.
- The threshold effect is defined as the minimum carrier to noise ratio that gives the output signal to noise ratio not less than the value predicted by the usual signal to noise formula assuming a small noise power.



Representation of equation using phasor diagram

The condition required to occur clicks are

Conditions for positive clicks:

$$r(t) > A_c$$

$$\psi(t) < \pi < \psi(t) + d\psi(t)$$

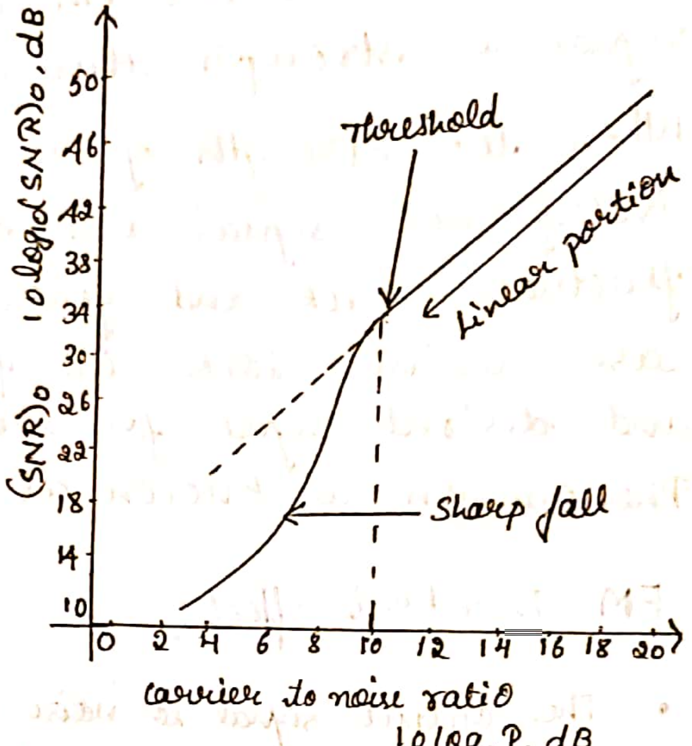
$$\frac{d\psi(t)}{dt} > 0$$

Conditions for negative clicks:

$$r(t) > A_c$$

$$\psi(t) > -\pi > \psi(t) + d\psi(t)$$

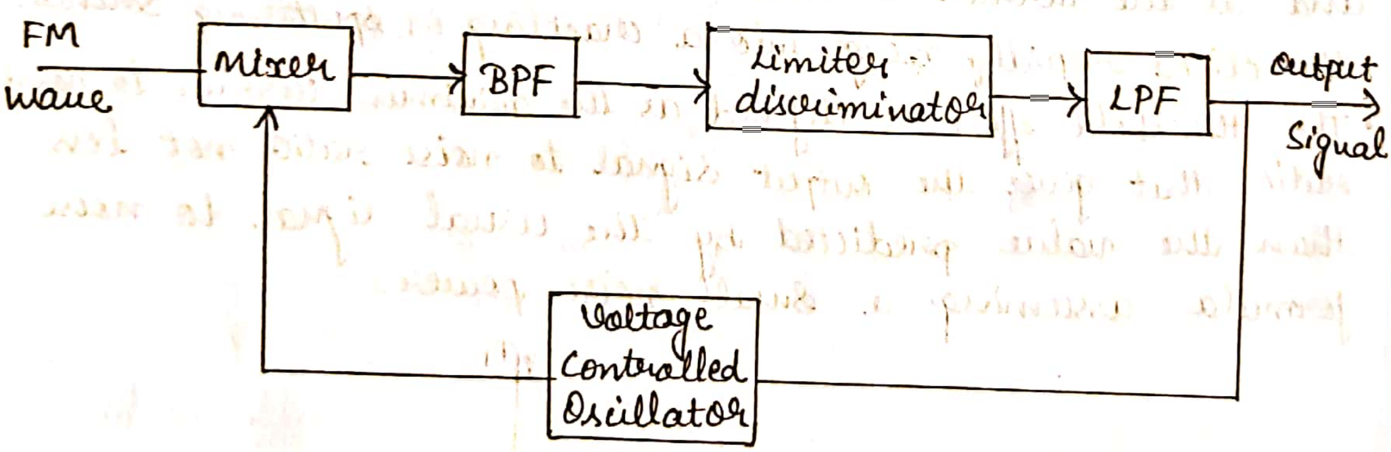
$$\frac{d\psi(t)}{dt} < 0$$



Graph representing the relationship b/w P and $(SNR)_o$

FM Threshold Reduction:

In specific applications such as space communication lesser threshold in a FM receiver is required. For such applications, FM threshold can be reduced by using FM demodulator with negative feedback known as FMFB demodulator.



FMFB demodulator

Block diagram of FMFB demodulator is shown in above fig. From the figure the feedback VCO is connected and is controlled by the demodulated signal.

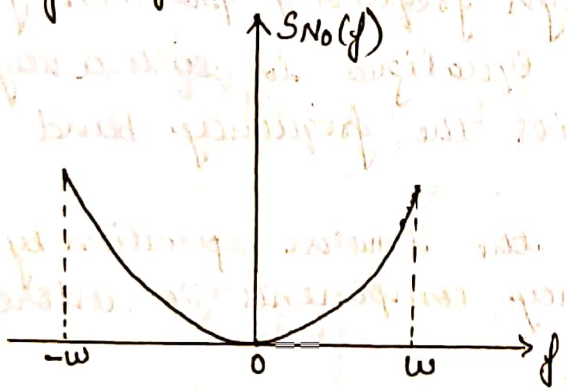
FMFB demodulator is essentially a tracking filter that can track only the slowly varying frequency of WBFM and consequently it responds only to a narrow band of noise centered about the instantaneous carrier frequency. As a result, FMFB receivers allow a threshold extension.

Like the FMFB demodulator, the PLL is also a tracking filter and hence it also provides threshold extension.

Pre-Emphasis and De-Emphasis in FM:

The power spectral density of the noise at the receiver output is shown in fig (a), it increases rapidly with frequency.

The power spectral density of a typical message source is shown in fig (b), it usually falls off appreciably at higher frequencies.



fig(a): PSD of output noise in FM

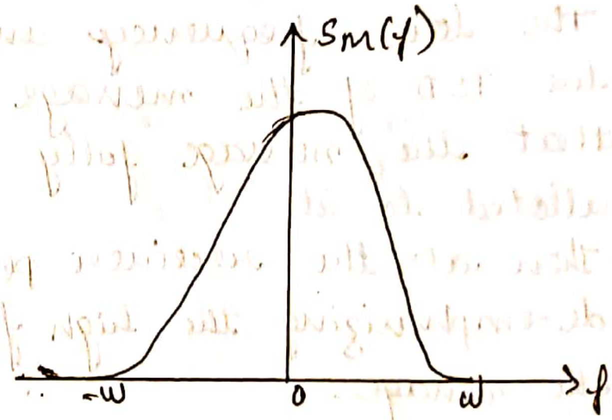
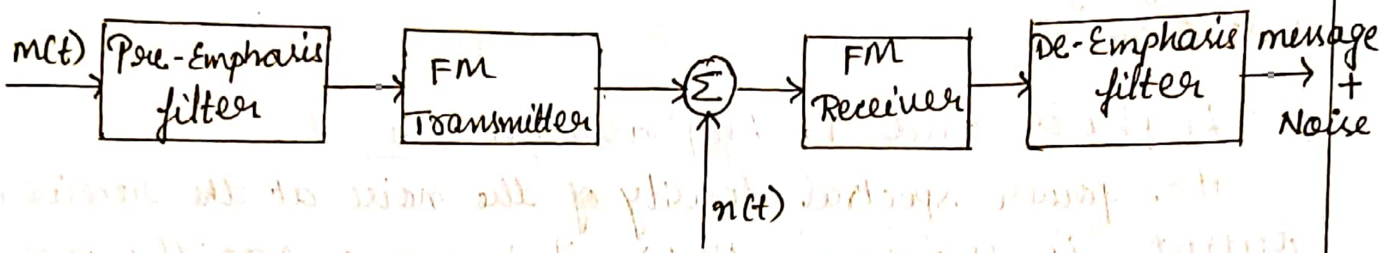


fig (b): PSD of message signal.

Thus at $f = \pm W$, the relative spectral density of the message is quite low, whereas that of the output noise is high in comparision. The message signal is not using the frequency band allowed to it in an efficient manner.

i) One way of improving the noise performance of the system is to slightly reduce the bandwidth of the LPF to reject large amount of noise power while losing only a small amount of message power. But this approach is not satisfactory because the distortion of the message

ii) A more satisfactory approach is the use of pre-emphasis in the transmitter and de-emphasis in the receiver as shown in below figure



- In this method, we artificially emphasize the high frequency components of the message signal prior to modulation in the transmitter before the noise is introduced in the receiver.
- The low frequency and high frequency portions of the PSD of the message are equalized in such a way that the message fully occupies the frequency band alloted to it.
- Then at the receiver perform the inverse operation by de-emphasizing the high frequency components to restore the message.
- In this process, the high frequency noise is reduced thereby effectively increasing the output SNR of the system.

Pre-emphasis has the transfer function

$$H_{pre}(f) = k\left[1 + j\, f/f_1\right]$$

The corresponding de-emphasis network in the receiver will have a transfer function

$$H_{de}(f) = \frac{1}{H_{pre}(f)} = \frac{1}{k\left[1 + j(f/f_1)\right]}$$

Constant K is chosen such that the average power of the pre-emphasized modulating signal be the same as the average power of the original modulating signal.

Derivation of improvement in SNR due to pre-emphasis and de-emphasis

Note: To derive 'K'

Let the power spectral density of the original modulating signal be

$$S_m(f) = \begin{cases} \frac{1}{1+(f/f_1)^2} & |f| < W \\ 0 & \text{elsewhere} \end{cases}$$

Average power of pre-emphasized signal = Average power of the original signal.

$$\int_{-\infty}^{\infty} |H(f)|^2 S_m(f)\, df = \int_{-\infty}^{\infty} S_m(f)\, df$$

$$\int_{-\infty}^{\infty} K^2 \cancel{\left(1+\frac{f^2}{f_1^2}\right)} \frac{1}{\cancel{(1+f^2/f_1^2)}}\, df = \int_{-\infty}^{\infty} \frac{1}{1+(f/f_1)^2}\, df$$

$$\int_{-W}^{W} K^2\, df = \int_{-W}^{W} \frac{1}{1+(f/f_1)^2}\, df$$

$f/f_1 = u$

$f = u f_1$

$df = f_1\, du$

$$K^2 [f]_{-W}^{W} = \int_{-W}^{W} \frac{f_1\, du}{1+u^2}$$

$$2WK^2 = f_1 \left[\tan^{-1}(f/f_1)\right]_{-W}^{W}$$

$$2WK^2 = 2 f_1 \tan^{-1}(W/f_1)$$

$$\boxed{K^2 = \frac{f_1}{W} \tan^{-1}\left[W/f_1\right]}$$

The improvement in o/p SNR due to pre-emphasis and de-emphasis in the transmitter and receiving end as

$$I = \frac{\text{Average output noise power without pre-emphasis \& de-emphasis}}{\text{Average output noise power with pre-emphasis and de-emphasis}}$$

The Average output noise power without pre-emphasis and de-Emphasis $= \frac{2}{3}\frac{N_0 W^3}{A_c^2}$

Average output noise power with pre-emphasis and de-emphasis filter is given by, $\int_{-W}^{W} S_{No}(f)\,|H_{de}(f)|^2\,df$

where, $S_{No}(f) = \frac{N_0 f^2}{A_c^2}$

$$= \int_{-W}^{W} \frac{N_0 f^2}{A_c^2} \frac{1}{k^2[1+(f/f_1)^2]} df$$

Hence

$$I = \frac{\frac{2}{3}\frac{N_0 W^3}{A_c^2}}{\frac{N_0}{A_c^2 k^2}\int_{-W}^{W} f^2\left[\frac{1}{1+(f/f_1)^2}\right] df}$$

$$I = \frac{2W^3}{\frac{3}{k^2}\underbrace{\int_{-W}^{W}\frac{f^2}{1+(f/f_1)^2} df}}$$

Since $\int \frac{x^2\,dx}{a^2+b^2x^2} = \frac{x}{b^2} - \frac{a}{b^3}\tan^{-1}\left(\frac{bx}{a}\right)$

$$I = \frac{2W^3}{\frac{3}{k^2}\left[f f_1^2 - f_1^3 \tan^{-1}(f/f_1)\right]_{-W}^{W}}$$

$$I = \frac{2w^3}{3/k^2\left[2wf_1^2 - 2f_1^3 \tan^{-1}(w/f_1)\right]}$$

$$I = \frac{w^3}{3/k^2\left[wf_1^2 - f_1^3 \tan^{-1}(w/f_1)\right]}$$

W.K.T $\quad k^2 = f_1/w \tan^{-1}(w/f_1)$

$$I = \frac{w^3}{\frac{3w}{f_1 \tan^{-1}(w/f_1)}\left[wf_1^2 - f_1^3 \tan^{-1}(w/f_1)\right]}$$

$$I = \frac{w^2}{3\left[\frac{wf_1^2}{f_1 \tan^{-1}(w/f_1)} - \frac{f_1^3 \cancel{\tan^{-1}(w/f_1)}}{f_1 \cancel{\tan^{-1}(w/f_1)}}\right]}$$

$$I = \frac{w^2}{3\left[\frac{wf_1}{\tan^{-1}(w/f_1)} - f_1^2\right]}$$

$$I = \frac{w^2}{3\left[\frac{wf_1 - f_1^2 \tan^{-1}(w/f_1)}{\tan^{-1}(w/f_1)}\right]}$$

$$I = \frac{w^2 \tan^{-1}(w/f_1)}{3\left[wf_1 - f_1^2 \tan^{-1}(w/f_1)\right]}$$

Divide by $1/f_1^2$

$$I = \frac{\left(\frac{w}{f_1}\right)^2 \tan^{-1}\left(\frac{w}{f_1}\right)}{3\left[\left(\frac{w}{f_1}\right) - \tan^{-1}\left(\frac{w}{f_1}\right)\right]}$$

For commercial broadcasting, typical values are

$f_1 = 2.1\,KHz$, $w = 15\,KHz$

$$I = \frac{\left(\frac{15}{2.1}\right)^2 \tan^{-1}\left(\frac{15}{2.1}\right)}{3\left[\left(15/2.1\right) - \tan^{-1}\left(15/2.1\right)\right]}$$

$$I = 4.2633$$

In decibel $\Rightarrow I_{dB} = 10\log_{10} 4.2633$

$$I_{dB} = 6.3\,dB$$

Thus by using the simple pre-emphasis and de-emphasis, a significant improvement in the noise performance of the receiver is obtained.

Module - 5

Digital Representation of Analog Signals

Why Digitize Analog Source?

* Digital systems are less sensitive to noise than analog
* With digital systems, it is easier to integrate different services, for example, video and the accompanying soundtrack, into the same transmission scheme.
* Various media sharing strategies, known as multiplexing techniques, are more easily implemented with digital transmission strategies.
* Analog data cannot be analyzed on Digital computer.
* Analog data cannot be compressed efficiently.
* Analog data transmission do not have efficient error detection and correction techniques.
* Circuitry for handling digital signals is easier to repeat and digital circuits are less sensitive to physical effects such as vibration and temperature.

Sampling:

- To convert an analog signal into digital data. Sampling quantization and encoding has to be performed.
- An analog signal is first sampled, which can be defined a "the process of converting an analog signal into discreet time signal by measuring the signals at periodic instan of time".

Sampling theorem:

Statement:

"If a finite energy signal g(t) contains no frequencies higher than 'W' hertz, it can be completely recovered from i ordinates of a sequence points spaced '$1/2W$' seconds apart".

Proof:

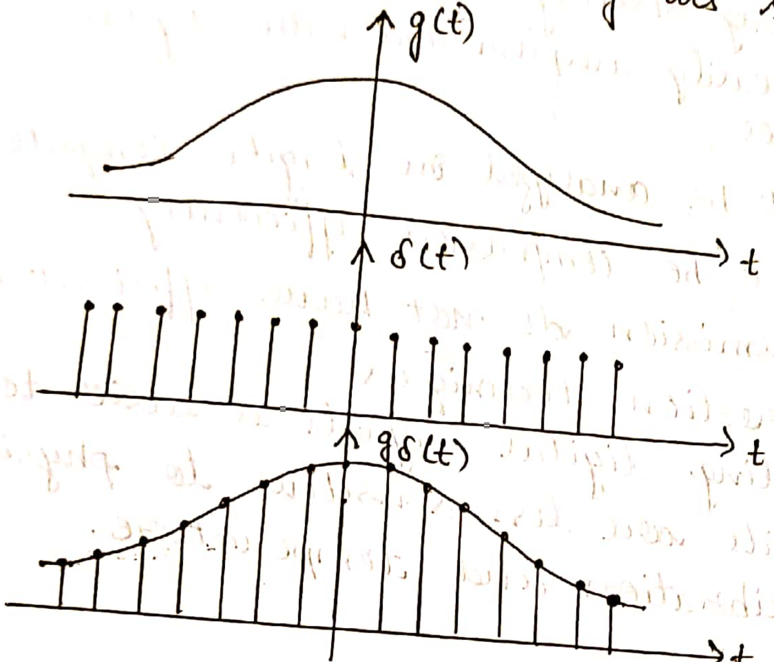
The sampling theorem for band limited signal (can ...) proved in two parts.

1) Representing g(t) in terms of its samples

2) Reconstructing it from its samples.

To prove this, let us consider an analog signal of finite energy and infinite duration as shown in figure g(t)

. Representing g(t) in terms of its samples $g(nT_s)$



Let us consider the sample values of g(t) at $t=0, \pm T_s, \pm 2T_s, \pm 3T_s$ .... Therefore the entire series can be denoted as $g(nT_s)$, where $n = 0, \pm 1, \pm 2, \pm 3 ....$

Where T_s = sampling period, $f_s = \frac{1}{T_s}$ = sampling rate

To obtain $g_\delta(t)$ from g(t), we need to multiply g(t) with δ function.

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s) \longrightarrow (1)$$

where,

$g(nT_s)$ is samples of g(t)

$\delta(t - nT_s)$ indicates the samples placed at $\pm T_s$, $\pm 2T_s$, $\pm 3T_s$ .... & so on.

(1) is graphically represented in above figure

...ce $g(nT_s)$ are samples of $g(t)$

$$g(t)\delta(t-nT_s) = g(nT_s)\delta(t-nT_s)$$

Eqn (1) can be written as,

$$g_\delta(t) = g(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s) \longrightarrow (2)$$

W.K.T multiplication of 2 time function is equivalent to the convolution of their respective fourier transforms

$$g(t) \to G(f)$$

$$g_\delta(t) \to G_\delta(f)$$

$$\delta(t-nT_s) \to f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s)$$

$$\therefore G_\delta(f) = FT\{g(t) * FT\,\delta(t-nT_s)\}$$

$$G_\delta(f) = G(f) * f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s)$$

$$\left\langle G_\delta(f) = f_s\sum_{n=-\infty}^{\infty} G(f) * \delta(f-nf_s)\right\rangle \longrightarrow (3)$$

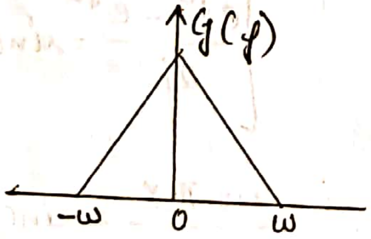
From the properties of delta function, Eqn (3) becomes

$$\left\langle G_\delta(f) = f_s\sum_{n=-\infty}^{\infty} G(f-nf_s)\right\rangle \longrightarrow (4)$$

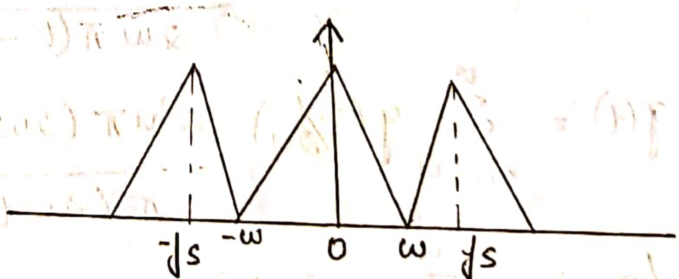
The spectrum of the above $g(t)$ Expression assuming $f_s = 2W$ and we know that $\delta(t-nT_s) \to e^{-j2\pi nfT_s}$
$= e^{-j\omega nT_s}$

Taking FT of Eqn (1), becomes

$$\left\langle G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s)\,e^{-j2\pi nfT_s}\right\rangle \longrightarrow (5)$$



Spectrum of g(t)



Spectrum of $g_\delta(t)$ for $f_s = 2W$

Reconstruction of signal from it's samples

Let us consider $f_s = 2W$, $T_s = 1/2W$ $\quad [T_s = 1/f_s = 1/2W]$

$\therefore$ Eqn (5) implies

$$\left\langle G_\delta(f) = \sum_{n=-\infty}^{\infty} g(n/2W)\,e^{(-j\pi nf/W)}\right\rangle \longrightarrow (6)$$

substituting $f_s = 2W$ in Eq^n (4)

$$G_\delta(f) = f_s\, G(f)$$

$$G_\delta(f) = 2W\, G(f)$$

$$\therefore \left\langle G(f) = \tfrac{1}{2W}\, G_\delta(f) \right\rangle \rightarrow (7)$$

substituting Eq^n (6) in (7)

$$\left\langle G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g(n/2W)\, e^{-j\pi n f/W} \right\rangle \rightarrow (8)$$

Taking inverse FT of $G(f)$

$$\left\langle g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f t}\, df \right\rangle \longrightarrow (9)$$

$$g(t) = \int_{-\infty}^{\infty} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g(n/2W)\, e^{-j\pi n f/W}\, e^{j2\pi f t}\, df$$

Interchange the order of summation and integration

$$g(t) = \sum_{n=-\infty}^{\infty} g(n/2W)\, \frac{1}{2W} \int_{-W}^{W} e^{j2\pi f(t - n/2W)}$$

$$g(t) = \sum_{n=-\infty}^{\infty} g(n/2W)\, \tfrac{1}{2W} \left. \frac{e^{j2\pi f(t-n/2W)}}{j2\pi(t - n/2W)} \right]_{-W}^{W} \qquad \int e^{af} = \frac{e^{af}}{a}$$

$$g(t) = \sum_{n=-\infty}^{\infty} g(n/2W)\, \tfrac{1}{2W} \left[\frac{e^{j2\pi W(t-n/2W)} - e^{-j2\pi W(t-n/2W)}}{j2\pi(t - n/2W)} \right]$$

$$g(t) = \sum_{n=-\infty}^{\infty} g(n/2W)\, \frac{\sin 2\pi W(t - n/2W)}{2W\pi(t - n/2W)} \qquad \left[\frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin\theta\right.$$

$$g(t) = \sum_{n=-\infty}^{\infty} g(n/2W)\, \frac{\sin \pi(2Wt - n)}{\pi(2Wt - n)} \qquad \frac{\sin \pi x}{\pi x} = \text{sinc}\, x$$

$$\left\langle g(t) = \sum_{n=-\infty}^{\infty} g(n/2W)\, \text{sinc}(2Wt - n) \right\rangle \longrightarrow (10)$$

Eq^n (10) is called an interpolation formula for reconstruction of the original signal $g(t)$.

A continuous time signal can be completely represented in its samples and recovered back if the sampling frequency is twice of the highest frequency content of the signal i.e

$$f_s \geq 2W$$

where, f_s = sampling frequency
W = Higher frequency content of message signal.

ii) <u>Nyquist rate</u>:

When the sampling rate becomes exactly equal to 2W samples/sec for a given Bandwidth of W Hertz then it is called Nyquist rate.

$$\text{Nyquist rate} = 2W \text{ Hz}.$$

<u>Nyquist Interval</u>:

It is the time interval between any two adjacent samples when sampling rate is Nyquist rate.

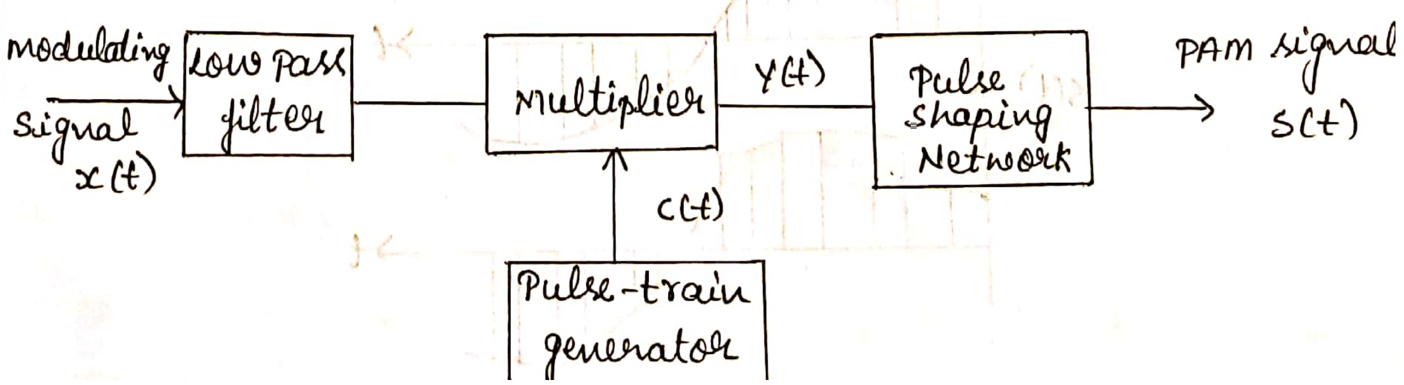
$$\text{Nyquist interval} = \frac{1}{2W} \text{ seconds}.$$

Pulse Amplitude Modulation:

The process in which the amplitude of the pulse is varied with respect to amplitude of the modulating signal at the sampling instant keeping width and position of pulse constant is known as pulse amplitude modulation (PAM).

<u>Generation of PAM</u>:

Block diagram for PAM generation is shown in figure



• The modulating signal $x(t)$ is bandlimited to freq- 'W' Hz by passing through LPF of frequency W Hz. Low pass filtering also avoids aliasing.

• The bandlimited signal is then sampled at the multiplier. Multiplier samples $x(t)$ with the help of pulse-train generator signal $c(t)$ as shown in figure. multiplication of $x(t)$ and $c(t)$ produces PAM signal $y(t)$.

• The pulse shaping Network produces the flat top pulses as shown in fig. This is the required PAM signal.

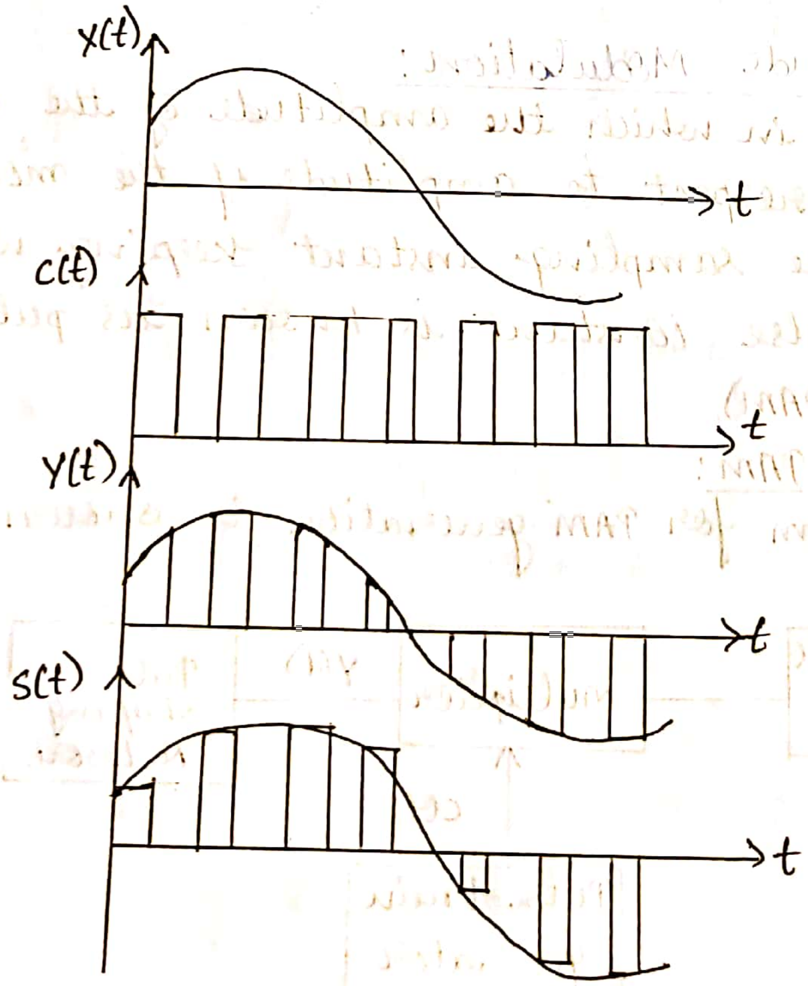
Note: Flat top sampling:

The top of the samples remain constant and it is equal to instantaneous value of baseband signal $x(t)$ at the start of sampling.

By using flat-top samples to generate a PAM signal, amplitude distortion occurs.

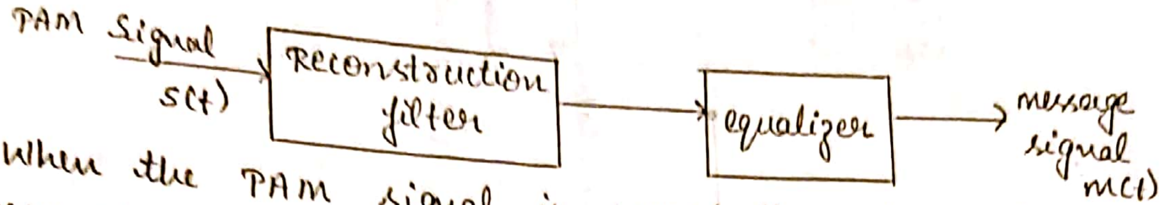
The distortion caused by the use of PAM to transmit an analog information bearing signal is referred to as the aperture effect.

Waveforms:



...tion of PAM:

Reconvering of m(t) from PAM signal s(t) is known ...s detection.



- When the PAM signal is passed through a lowpass reconstruction filter, the filter reconstructs the analog signal from PAM pulses.
- Equalizer in cascade with lowpass reconstruction filter compensates the aperature effect and produces the message signal m(t).

Advantages:

- PAM can be easily generated and detected.

Disadvantages:

- Bandwidth required for PAM transmission is larger than the maximum frequency of message signal [i.e W Hz]
- Interface of noise is maximum, because the amplitud of PAM pulses varies according to message signal.

Applications:

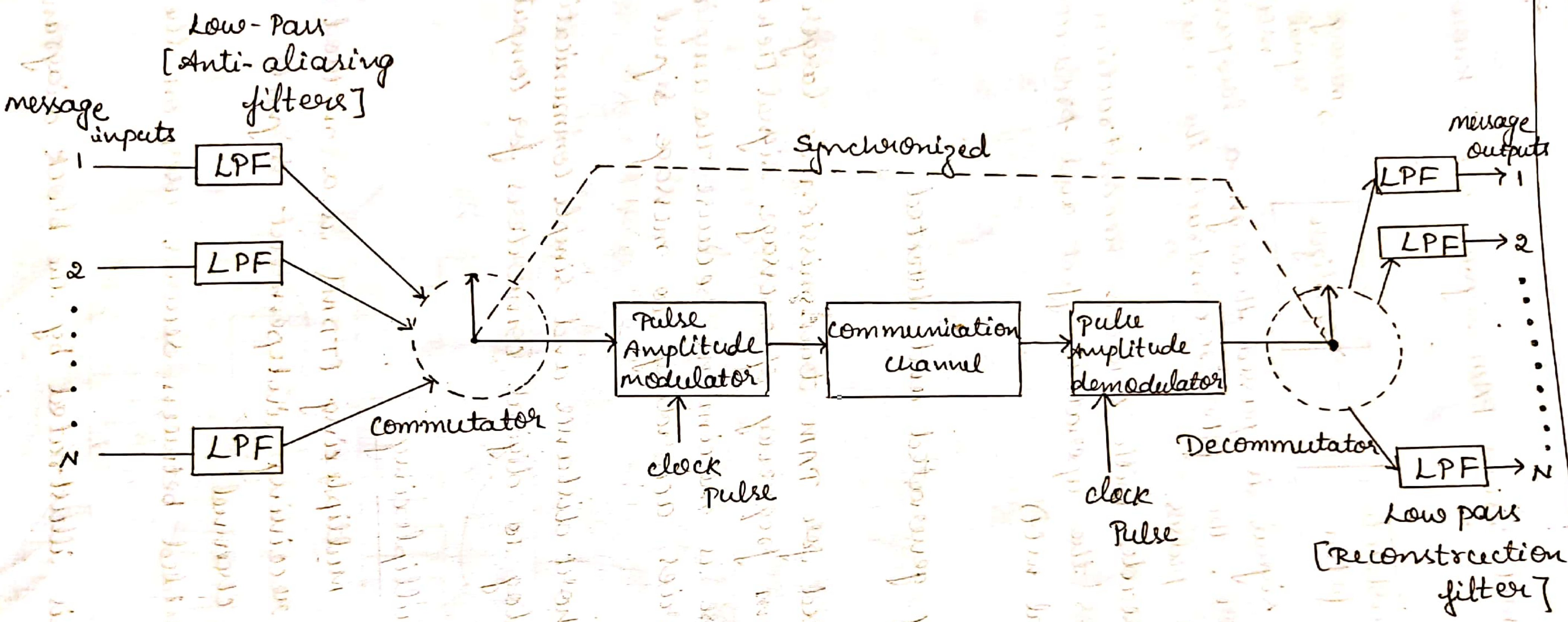
- PAM is used for short distance and simple communicatio
- It is used as Analog to digital converters for compute interfacing.

Time Division multiplexing:

Time Division multiplexing [TDM] is a method transmitting and receiving independent signals over a common channel.

channel is divided between several user on the basis of time.

The concept of TDM is illustrated by the block diagra shown in figure.



Block Diagram of TDM System

Each input signal is first restricted in Bandwidth by a low-pass filter to remove the frequencies that are non essential.

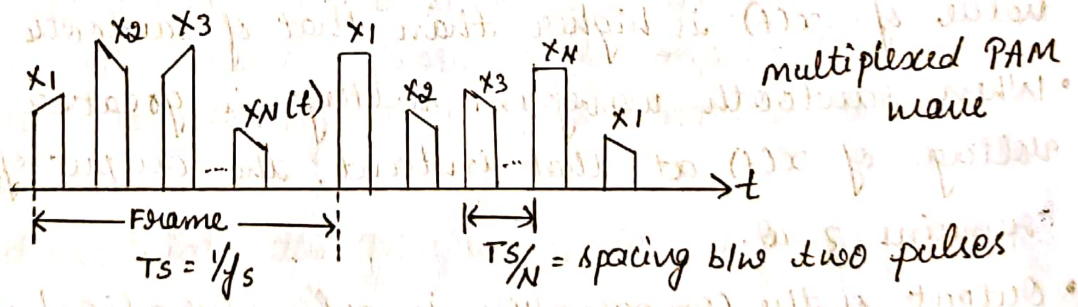
- The pre-aliasing filter output are then applied to a commutator which consists of electronic switch.

The functions of the commutator is

1) To take a narrow samples of each of the N input messages at a rate f_s that is slightly higher than 2W.
2) To sequentially interleave these N samples inside the sampling interval $T_s = 1/f_s$.

- After the commutation process, the multiplexed signal is applied to a pulse modulator, the purpose of which is to transform the multiplexed signal into a form suitable for transmission over the common channel.
- At the receiving end, pulse amplitude demodulator is used to perform the reverse operation of PAM.
- Decommutator picks the samples of incoming signal and distributes to appropriate low-pass reconstruction filter. Decommutator operates in synchronism with the commutator in the transmitter.

Note:



- Spacing between two pulses $= T_s/N$
- Signalling rate / transmission rate / bit rate $\gamma = \dfrac{1}{\text{spacing b/w two pulses}}$

$$\gamma = \frac{1}{T_s/N}$$

$$\boxed{\gamma = N f_s}$$

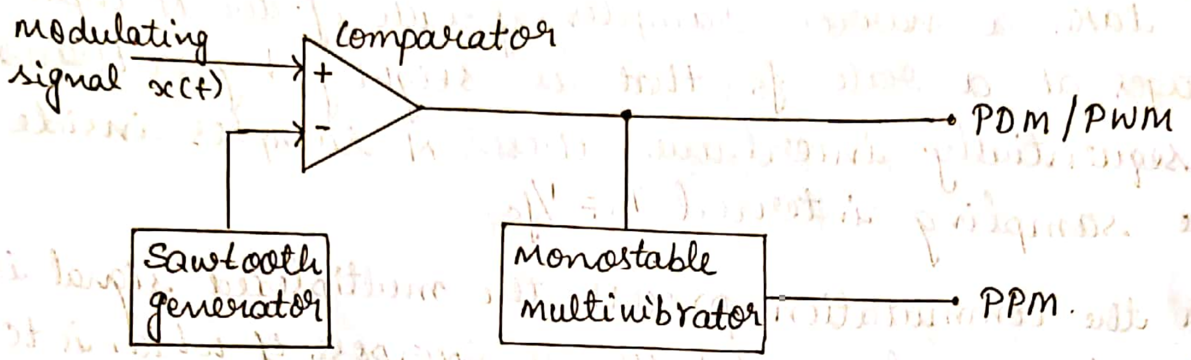
- Minimum transmission Bandwidth of TDM channel is

$$(BW)_T = NW$$

Pulse Position Modulation:

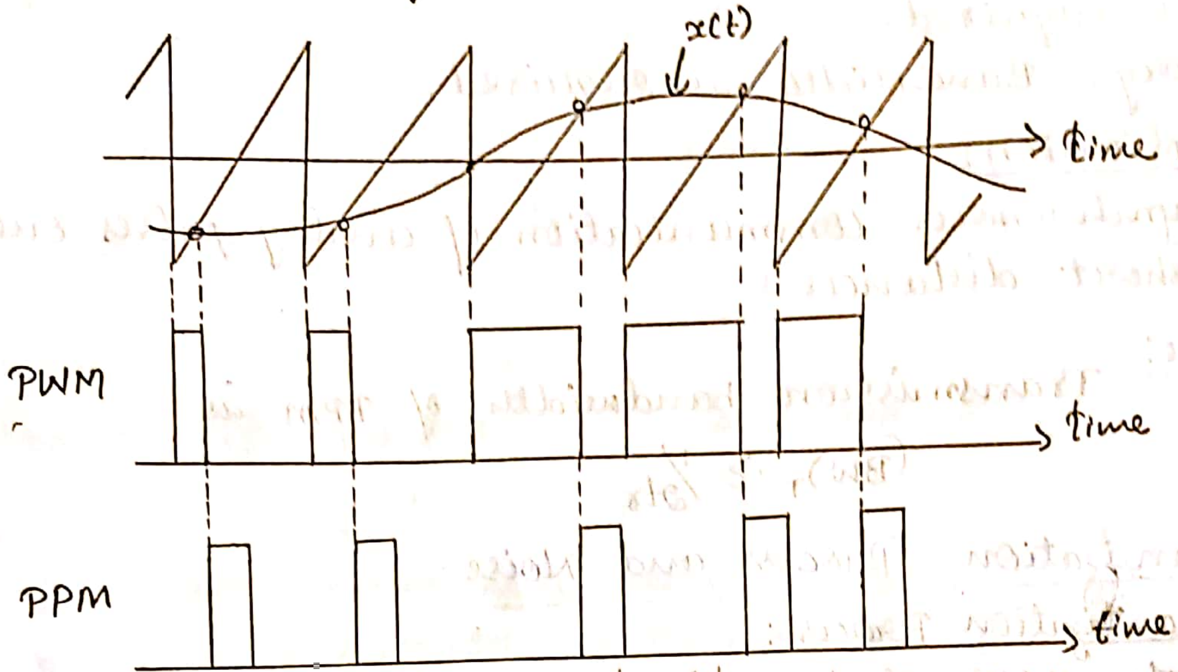
It is a type of pulse modulation, in which the position of each pulse is varied with respect to the amplitude of modulating signal keeping amplitude and width of the pulse constant.

Generation of PPM:



- The block diagram to generate PPM is shown in above figure. The scheme combines both sampling and modulation operation.
- Sawtooth generator generates sawtooth signal of frequency f_s and it is applied to the inverting input of comparator.
- Modulating signal $x(t)$ is applied to the non-inverting input of the comparator as shown in figure.
- Output of the comparator is high when instantaneous value of $x(t)$ is higher than that of sawtooth waveform.
- When sawtooth waveform voltage is greater than voltage of $x(t)$ at that instant, the output of comparator remains zero.
- Output of the comparator is pulse duration/width modulation as shown in waveform.
- To generate PPM, PDM/PWM signal is used as the trigger input to one monostable multivibrator.
- The monostable output remains zero until it is triggered the monostable is triggered on the falling edge of PDM. the output of monostable then switches to positive

turation level. This voltage remains high for the fixed period then goes low.



Detection of PPM waves:

To get back original signal m(t) from s(t) PPM receiver may proceed as follows:

- Convert the received PPM wave into PWM wave with the same modulation.
- Integrate this PWM wave using a device with a finite integration time.
- Sample the output of the integrator at a uniform rate to produce a PAM wave, whose pulse amplitudes are proportional to the signal samples $x(nT_s)$ of the original PPM wave s(t).
- Finally, demodulate the PAM wave to recover the message signal m(t).

Advantages:

- In PWM amplitude is held constant thus less noise interference.
- Signal and noise seperation is very easy.
- Due to constant pulse widths and amplitudes, transmission power for each pulse is same.

Disadvantages:

- Synchronization between transmitter and receiver is required.
- Large Bandwidth is required.

Application:

- Synchronous communication of analog pulses over short distances.

Note: Transmission bandwidth of PPM is

$$(BW)_T \geq 1/2t_r$$

Quantization Process and Noise:

Quantization Process:

The process of transforming the sample amplitude $x(nT_s)$ of a message signal $x(t)$ into a discrete amplitude $v(nT_s)$ value is referred as quantizing process.

continuous sample $x(nT_s)$ → Quantizer → Discrete samples $v(nT_s)$

Quantization can be broadly classified into uniform quantization and non uniform quantization.

In uniform quantization the difference between two quantization levels (step size) remains constant over the complete amplitude range otherwise it is non-uniform quantization.

Types of uniform Quantizers:

1) Mid-Rise type Quantizer
2) Mid-Tread type Quantizer.

In the stair case like graph, the origin lies the middle of the tread portion in mid-Tread type where as the origin lies in the middle of the rise portion in the mid-Rise type.

An anti-aliasing filter is basically a filter used to ensure that the input signal to sampler is free from the unwanted frequency components. LPF remove the frequencies greater than 'W' before sampling.

Sampling:

The incoming message signal is sampled by passing through the sampler. In order to ensure perfect reconstruction of the message signal at the receiver, the sampling rate must be greater than twice the highest frequency components 'W' of the message signal.

Quantization:

The process of transforming sampled amplitude values of a message signal into a discrete amplitude value is referred to as Quantization.

Encoding:

After sampling and quantizing, discrete set of values are not in the form best suited to transmission over a line or radio path. Thus to make the signal more robust to noise, interference and other channel degradations, Encoding has to be carried out.

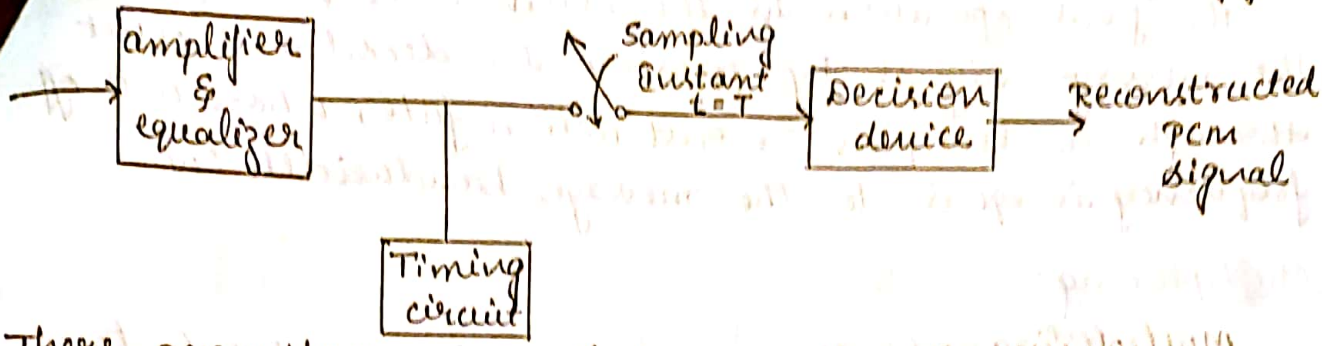
Encoding process translates the discrete set of sample value to a more appropriate form of binary signal.

Regenerative Repeater:

When the PCM signal is transmitted over the channel, it get distorted due to channel noise. Hence regenerative repeater are used at regular spacings to reconstruct the PCM signal back.

Regenerative repeaters receives the noisy PCM signal, performs amplification and equalization on it and constructs new PCM signal.

...ck diagram of regenerative repeater is shown in figure



There are three main blocks

1) Equalizer and amplifier
2) Timing circuit
3) Decision Device

The distorted PCM signal is amplified by amplifier. The equalizer shapes the received pulses so as to compensate for the effects of amplitude and phase distortion produced by the transmission characteristics of the channel.

The timing circuit derives the sampling instant t=T, where the signal to noise ratio is maximum.

The decision device compares the sampled PCM signal with threshold. If the threshold is exceeded, the new binary '1' is generated, otherwise binary '0' is sent. Thus the new PCM signal is generated, which is totally free of noise.

Problems associated with regenerative repeater.

1) If the channel noise is too high, then binary '1' is treated as binary '0' and vice-versa.
2) The spacing between the pulses is not exactly same as that was transmitted. This is called timing jitter. It creates distortion in the regenerated PCM signal.

Decoding:

The first operation in the receiver is to regenerate the received pulses one last time. These pulses are then regrouped into codewords and decoded into a quantized PAM signal.

Filtering:

The final operation in the receiver is to recover the message signal by passing the decoder output through a low pass reconstruction filter whose cut-off frequency is equal to the message bandwidth 'W'.

Multiplexing:

The applications using PCM, it is natural to multiplex different message sources by time division.

As the number of independent message sources is increased, the time interval that may be alloted to each source has to be reduced, since all of them must be accomodated into a time interval equal to the reciprocal of the sampling rate.

Quantization Noise and signal to Noise ratio (SNR) in PCM:

Error introduced because of quantization is called quantization error.

$$\langle \varepsilon = x(nT_s) - v(nT_s) \rangle$$

The signal to quantization noise ratio is defined as

$$\frac{S}{N} = \frac{\text{average power of the signal}}{\text{average power of the quantization noise}}$$

average power of quantization Noise:

Since quantization noise 'ε' is random variable, average power of quantization noise is $E[\varepsilon^2]$ mean square value [if R=1].

W.K.T The mean square value of a random variable 'x' is

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f_x(x)\,dx$$

IIIly

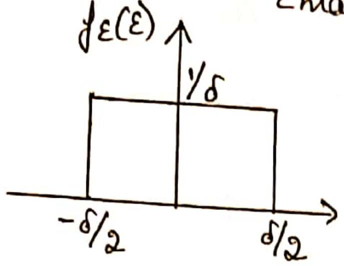
$$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\,d\varepsilon$$

(4)

Assume that the quantization noise 'ε' will be uniformly distributed random variable over the interval $[-\delta/2, \delta/2]$

The maximum quantization noise is

$$\varepsilon_{max} = |\delta/2| = \pm\delta/2$$



$$f_\varepsilon(\varepsilon) = \begin{cases} 1/\delta & -\delta/2 \le \varepsilon \le \delta/2 \\ 0 & \varepsilon > \delta/2 \end{cases}$$

$$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\, d\varepsilon$$

$$= \int_{-\delta/2}^{\delta/2} \varepsilon^2 \, 1/\delta \, d\varepsilon = 1/\delta \int_{-\delta/2}^{\delta/2} \varepsilon^2 \, d\varepsilon$$

$$= \frac{1}{\delta} \left. \varepsilon^3/3 \right]_{-\delta/2}^{\delta/2} = \frac{1}{3\delta}\left[\delta^3/8 + \delta^3/8\right]$$

$$E[\varepsilon^2] = \frac{1}{3\delta}\left[\delta^3/4\right] = \delta^2/12$$

$$\boxed{E[\varepsilon^2] = \delta^2/12}$$

Thus $\quad S/N = \dfrac{P}{\delta^2/12}$

$$(S/N) = \frac{12P}{\delta^2} \qquad \text{where, } \delta = \text{step size}$$

Let $x(nT_s)$ be of continuous sample amplitude ranges from $-x_{max}$ to $+x_{max}$.

$\therefore$ Total amplitude range $= x_{max} - (-x_{max}) = 2x_{max}$

If this amplitude range is divided into 'q' levels of quantizer, then the step size 'δ' is given as

$$\delta = \frac{2x_{max}}{q}$$

For Normalised signal

$$\delta = 2/q$$

If 'v' is the number of bits, the relationship between and 'q' are given by,

$$\langle q = 2^v \rangle$$

$$(S/N) = \frac{12P}{\left(\frac{2x_{max}}{q}\right)^2}$$

$$(S/N) = \frac{12Pq^2}{4x^2_{max}}$$

$$(S/N)_{normalized} = \frac{\overset{3}{\cancel{12}}Pq^2}{\cancel{4}}$$ For Normalized signal $x_{max} = 1$

$$(S/N) = 3Pq^2$$

$$(S/N) = 3P(2^v)^2 = 3P \cdot 2^{2v}$$

Now, to express (S/N) in decibels the above expression can be written as

$$(S/N)_{dB} = 10 \log_{10} (S/N) \, dB$$ where $P \leq 1$

$$= 10 \log (3 \times 2^{2v})$$

$$(S/N)_{dB} = 10 \log 3 + 10 (2v \log 2)$$

$$\langle (S/N)_{dB} = (4.78 + 6v) \, dB \rangle$$

<u>Note</u> :

- signalling rate / bit rate / bit per sample in PCM is
  $$\langle r = v f_s \rangle$$
- Transmission bandwidth of PCM is $(BW)_T \geq vW$
- average power of Quantization Noise $= \delta^2/12$
- Maximum signal to Noise ratio in PCM is

$$(S/N) = \frac{P}{\delta^2/12}$$

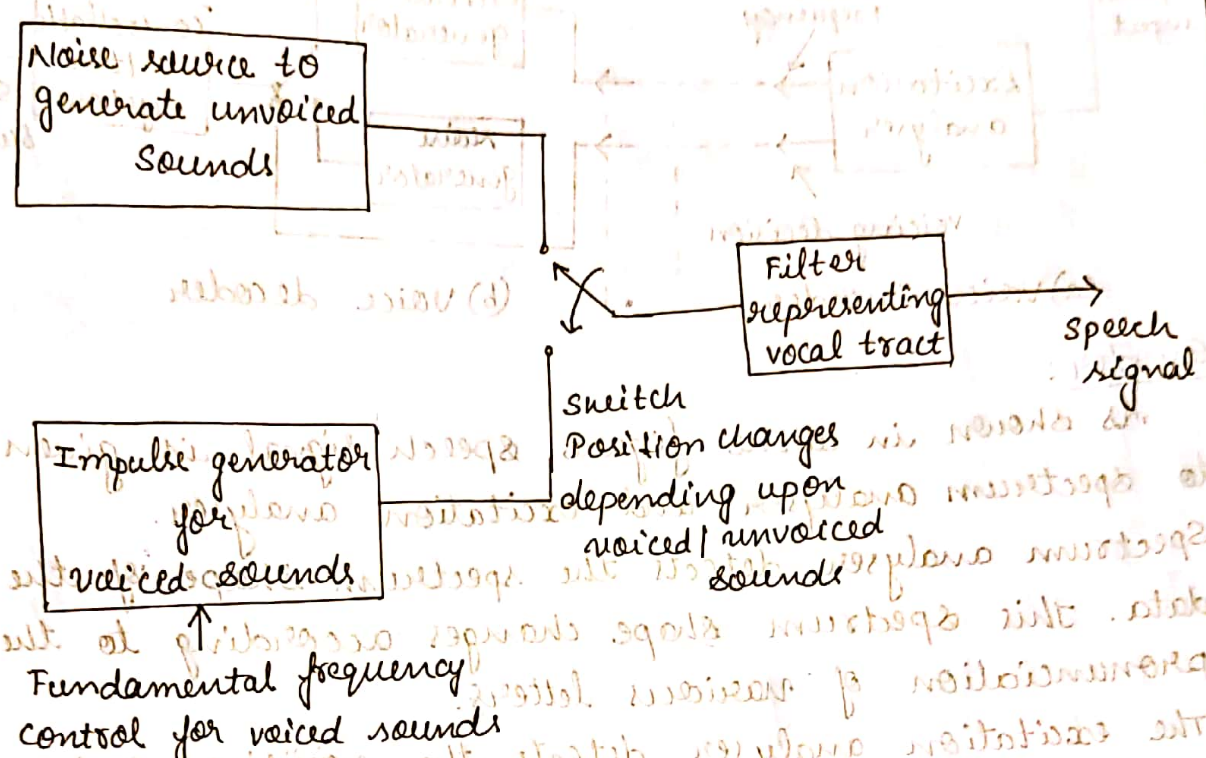
For $(S/N)_{normalized} = 3P \cdot 2^{2v}$

$$(S/N)_{dB} = (4.78 + 6v) \, dB$$

Application of vocoders:

The speech or voice coders are used for digital coding of speech. It is specially designed for coding speech signals. They operate at very low bit rate in the range of 1.2 to 2.4 Kbps

Voice model:



Voice model is as shown in above figure. It has two frequency sources.

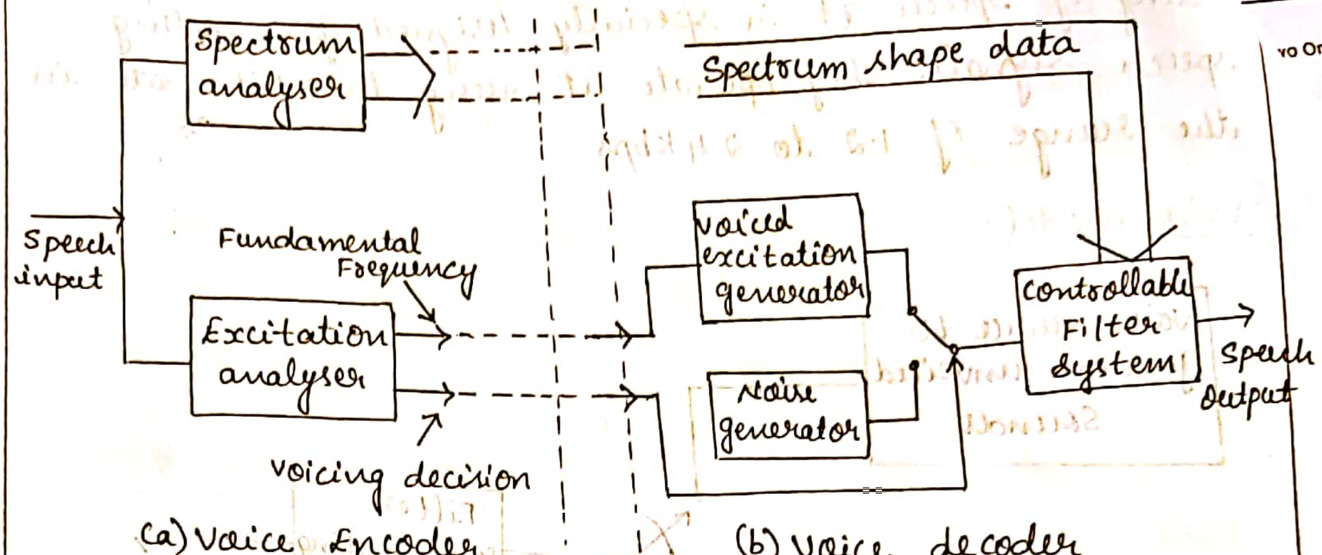
1) Frequency source used to generate unvoiced sounds when the speaker pronounces letter such as 's' & 'f'. A noise source is used to generate such unvoiced sound since their frequency spectrum is wide.

2) Voiced sound are simulated by impulse generator. Frequency is varied depending upon the pitch of the sound.

These voiced/unvoiced signals are then passed through a filter (vocal tract). In vocal tract, filtering is done with the help of tongue, lips, teeth etc.

Thus by using voice model, speech is generated.

Block diagram of vocoders:



(a) Voice Encoder (b) Voice decoder

Encoder:

As shown in above figure, speech signal is given to spectrum analyser and excitation analyser.

- Spectrum analyser detects the spectrum shape of the data. This spectrum shape changes according to the pronunciation of various letters.
- The excitation analyser detects the voicing decision. i.e whether speech is voiced/unvoiced. It also detects the fundamental frequency of voiced signals.
- Thus the encoder converts speech signal in terms of fundamental frequency, voicing decision and spectrum shape.

Decoder:

The decoder is used to generate speech from the data given by encoder.

- Decoder contains noise generator and voiced excitat generator frequency of voiced excitation generator is controlled depending upon fundamental frequency information from encoder.

- The voicing decision detects whether the sound is voiced / unvoiced.
- Controllable filter system, filters the signal generated from voiced excitation generator / noise generator.
- Frequency response of this filter is variable, it is changed according to spectrum shape data from the encoder. The output of the filter is the speech signal.